Jiangsu Normal University

Fundamentals of Physics
Lecture-07
Chap.12 Electricity

Junqing Zhao

junqingzhao@outlook.com

Tel: +86+0+13632729296

12-1~12-2 Electric Charge

- *The physics of electromagnetism* is at the root of computers, television, radio, telecommunications, household lighting, and even the ability of food wrap to cling to a container.
- This physics is also the basis of the natural world. Not only does it hold together all the atoms and molecules in the world, it also produces lightning, auroras, and rainbows.
- The physics of electromagnetism was first studied by the early *Greek philosophers*, who discovered that if a piece of amber is rubbed and then brought near bits of straw, the straw will jump to the amber. We now know that the attraction between amber and straw is due to an *electric force*.
- The Greek philosophers also discovered that if a certain type of stone (a naturally occurring magnet) is brought near bits of iron, the iron will jump to the stone. We now know that the attraction between magnet and iron is due to a *magnetic force*.
- From these modest origins with the Greek philosophers, the sciences of electricity and magnetism developed separately for centuries.

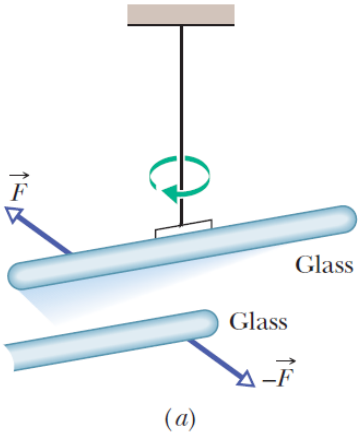
12-1~12-2 Electric Charge

- Until 1820, Hans Christian Oersted found a connection between the electricity and magnetism: an electric current in a wire can deflect a magnetic compass needle.
- The new science of electromagnetism was developed further by workers in many countries.
- One of the best was ***Michael Faraday***, a truly gifted experimenter with a talent for physical intuition and visualization. That talent is attested to by the fact that his collected laboratory notebooks do not contain a single equation.
- In the mid-nineteenth century, ***James Clerk Maxwell*** put Faraday's ideas into mathematical form, introduced many new ideas of his own, and put electromagnetism on a sound theoretical basis.

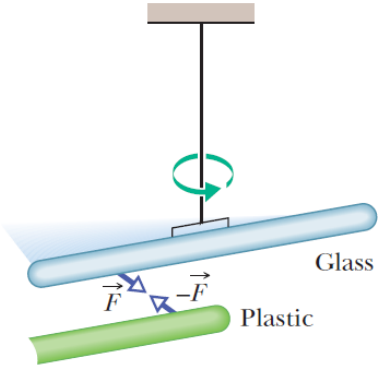
- We begin with electrical phenomena, and our first step is to discuss the nature of electric charge and electric force.

12-1~12-2 Electric Charge

- Here are two demonstrations that seem to be magic, but our job here is to make sense of them.
- After rubbing a glass rod with a silk cloth, we hang the rod by means of a thread tied around its center (Fig. a). Then we rub a second glass rod with the silk cloth and bring it near the hanging rod. The hanging rod magically moves away. We can see that a force repels it from the second rod, but how? There is ***no contact*** with that rod, no breeze to push on it, and no sound wave to disturb it.
- In the second demonstration we replace the second rod with a plastic rod that has been rubbed with fur. This time, the hanging rod moves toward the nearby rod (Fig. b). Like the repulsion, this attraction occurs ***without any contact or obvious communication*** between the rods.
- After a great many investigations, scientists figured out that the forces in these demonstrations are due to the ***electric charge*** that we set up on the rods when they are in contact with silk or fur.



(a)



(b)

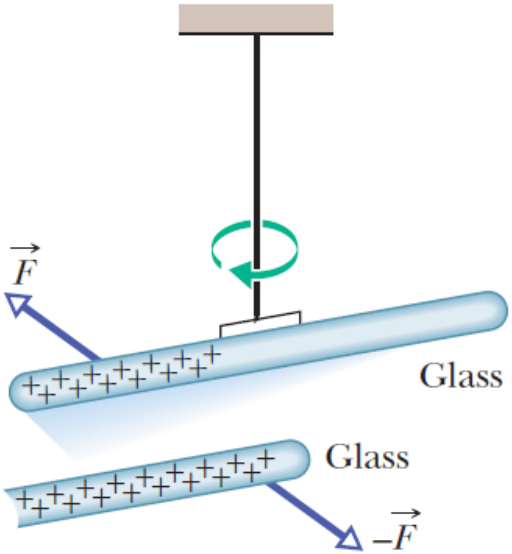
12-1~12-2 Electric Charge

- Electric charge is an ***intrinsic property*** of the fundamental particles that make up objects such as the rods, silk, and fur. That is, charge is a property that comes automatically with those particles wherever they exist.
- **Two Types:** There are two types of electric charge, named by Benjamin Franklin as ***positive charge*** and ***negative charge***. In most everyday objects there are equal numbers of negatively and positively charged particles, and so the net charge is zero. The charge is said to be *balanced*, and the object is said to be *electrically neutral* (or just neutral for short).
- Two of the grand mysteries in physics are (1) why does the universe have particles with electric charge (what is it, really?) and (2) why does electric charge come in two types (and not, say, one type or three types).We just do not know. Nevertheless, with lots of experiments scientists discovered that

Particles with the same sign of electrical charge repel each other, and particles with opposite signs attract each other.

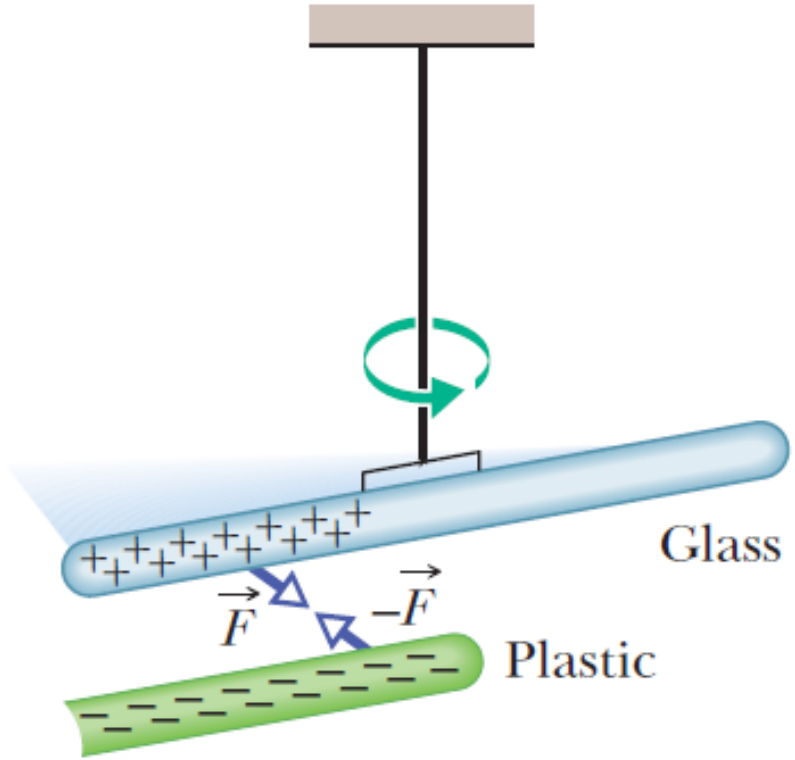
12-1~12-2 Electric Charge

- Now let's get back to the demonstrations to understand the motions of the rod as being something other than just magic.
- When we rub the *glass rod* with a *silk cloth*, a small amount of negative charge moves from the rod to the silk, leaving the rod with a small amount of ***excess positive charge***. We ***rub*** the silk over the rod to increase the number of contact points and thus the amount of transferred charge. We hang the rod from the thread so as to ***electrically isolate*** it from its surroundings (so that the surroundings cannot neutralize the rod by giving it enough negative charge to rebalance its charge).
- When we rub the second rod with the silk cloth, it too becomes positively charged. So when we bring it near the first rod, the two rods repel each other (Figure right).



12-1~12-2 Electric Charge

- Next, when we rub the *plastic rod* with *fur*, it gains ***excess negative charge*** from the fur.
- When we bring the plastic rod (with negative charge) near the hanging glass rod (with positive charge), the rods are attracted to each other (Figure below). All this is subtle. You cannot see the charge or its transfer, only the results.



12-1~12-2 Electric Charge

Conductors and Insulators

- We can classify materials generally according to the ability of charge to move through them.
- *Conductors* are materials through which charge can move rather freely; examples include metals (such as copper in common lamp wire), the human body, and tap water.
- *Nonconductors*—also called *insulators*—are materials through which charge cannot move freely; examples include rubber (such as the insulation on common lamp wire), plastic, glass, and chemically pure water.
- *Semiconductors* are materials that are intermediate between conductors and insulators; examples include silicon and germanium in computer chips.
- *Superconductors* are materials that are perfect conductors, allowing charge to move without any hindrance.

12-1~12-2 Electric Charge

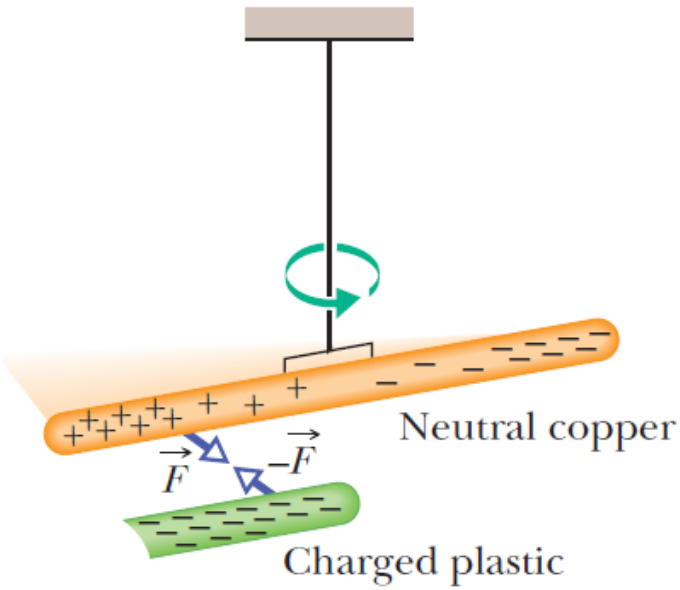
Charged Particles

- The properties of conductors and insulators are due to the structure and electrical nature of atoms. Atoms consist of ***positively charged protons***, ***negatively charged electrons***, and ***electrically neutral neutrons***. The protons and neutrons are packed tightly together in a central nucleus.
- The charge of a single electron and that of a single proton have the ***same magnitude but are opposite in sign***. Hence, an electrically neutral atom contains equal numbers of electrons and protons. Electrons are held near the nucleus because they have the electrical sign opposite that of the protons in the nucleus and thus are attracted to the nucleus. Were this not true, there would be no atoms and thus no you.
- When atoms of a conductor like copper come together to form the solid, some of their outermost (and so most loosely held) electrons become free to wander about within the solid, leaving behind positively charged atoms (positive ions).We call the mobile electrons ***conduction electrons***. There are few (if any) ***free electrons*** in a nonconductor.

12-1~12-2 Electric Charge

Induced Charge.

- The experiment of Figure right demonstrates the mobility of charge in a conductor. A negatively charged plastic rod will ***attract either end*** of an isolated neutral *copper rod*.
- What happens is that many of the conduction electrons in the closer end of the copper rod are repelled by the negative charge on the plastic rod, resulting in some of them moving to the far end of the copper rod, leaving the near end depleted in electrons and thus with an unbalanced positive charge. This positive charge is attracted to the negative charge in the plastic rod.
- Although the copper rod is still neutral, it is said to have an induced charge, which means that some of its positive and negative charges have been ***separated*** due to the presence of a nearby charge.



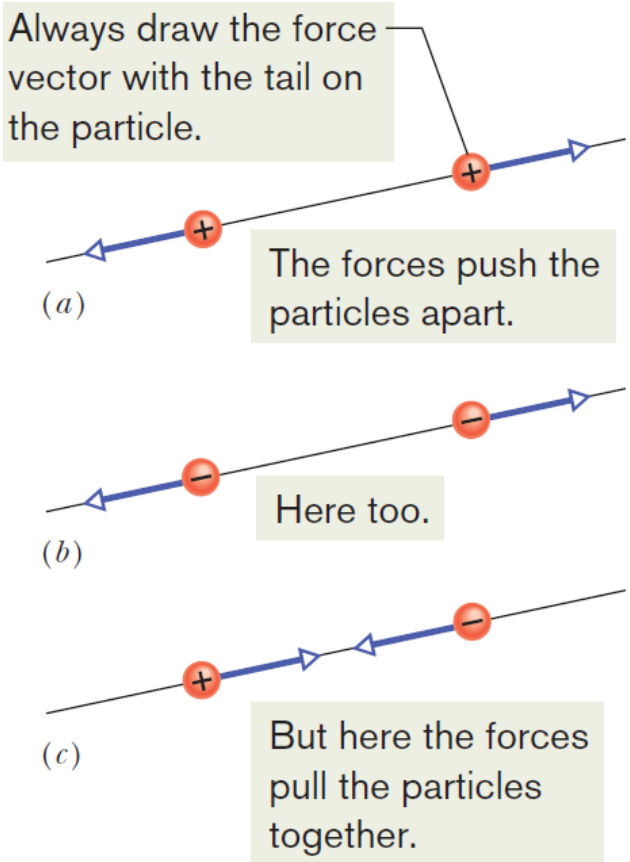
12-1~12-2 Electric Charge

- Similarly, if a positively charged glass rod is brought near one end of a neutral copper rod, induced charge is again set up in the neutral copper rod but now the near end gains conduction electrons, becomes negatively charged, and is attracted to the glass rod, while the far end is positively charged.
- Note that only conduction electrons, with their negative charges, can move; positive ions are fixed in place. Thus, an object becomes positively charged only through the *removal of negative charges*.

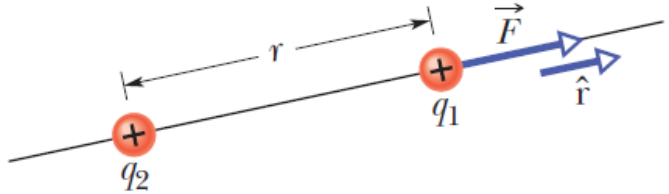
12-1~12-2 Electric Charge

Coulomb's Law

- Now we come to the equation for Coulomb's law. This equation works for only charged particles (and a few other things that can be treated as particles).
- For extended objects, with charge located in many different places, we need more powerful techniques. So, here we consider just charged particles.
- If two charged particles are brought near each other, they each exert an *electrostatic force* on the other. The direction of the force vectors depends on the signs of the charges. If the particles have the same sign of charge, they repel each other. The force vector on each is directly away from the other
- Particle (Figs. a and b). If, instead, the particles have opposite signs of charge, they attract each other. The force vector on each is directly toward the other particle (Fig. c).



12-1~12-2 Electric Charge



- The equation for the electrostatic forces acting on the particles is called **Coulomb's law** after Charles-Augustin de Coulomb, whose experiments in 1785 led him to it.
- Let's consider particle 1 with charge q_1 and particle 2 with charge q_2 , in Figure above. (These symbols can represent either positive or negative charge.) Let's also focus on particle 1 and write the force acting on it in terms of a unit vector $\hat{r}$ that points along a radial axis extending through the two particles, radially away from particle 2. With these decisions, we write the electrostatic force as
- $$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r} \quad \text{(Coulomb's law)}, \qquad (12\text{-}1)$$

where r is the separation between the particles and k is a positive constant called the electrostatic constant or the Coulomb constant.

12-1~12-2 Electric Charge

- Let's first check the direction of the force on particle 1 as given by Eq. 12-1. If q_1 and q_2 have the same sign, then the product $q_1 q_2$ gives us a positive result. So, Eq. 12-1 tells us that the force on particle 1 is in the direction of $\hat{\mathrm{r}}$.That checks, because particle 1 is being repelled from particle 2.
- Next, if q_1 and q_2 have opposite signs, the product $q_1 q_2$ gives us a negative result. So, now Eq. 12-1 tells us that the force on particle 1 is in the direction opposite .That checks because particle 1 is being attracted toward particle 2.

Unit

- The SI unit of charge is the coulomb. For practical reasons having to do with the accuracy of measurements, the coulomb unit is derived from the SI unit ampere for electric current i , which is the rate dq/dt at which charge moves past a point or through a region:
- $$i = \frac{dq}{dt} \quad \text{(electric current).} \qquad (12\text{-}2)$$
- Rearranging Eq. 12-2 and replacing the symbols with their units (coulombs C, amperes A, and seconds s) we see that

$$1\ \mathrm{C} = (1\ \mathrm{A})(1\ \mathrm{s}).$$

12-1~12-2 Electric Charge

Force Magnitude

- The electrostatic constant k in Eq. 12-1 is often written as $1/4\pi\varepsilon_0$. Then the magnitude of the electrostatic force in Coulomb's law becomes
- $$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \quad \text{(Coulomb's law).} \tag{12-3}$$
- $$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2}. \tag{12-4}$$
- The quantity ε_0, called the permittivity constant, sometimes appears separately in equations and is
- $$\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2}. \tag{12-5}$$

Multiple Force

- As with all forces in this textbook, the electrostatic force obeys *the principle of superposition*. Suppose we have n charged particles near a chosen particle called particle 1; then the net force on particle 1 is given by the vector sum
- $$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \cdots + \vec{F}_{1n} \tag{12-6}$$

12-1~12-2 Electric Charge

Shell Theories

Shell theory 1. A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center.

Shell theory 2. A charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell.

- **Note:** In the first theory, we assume that the charge on the shell is much greater than the particle's charge. Thus the presence of the particle has negligible effect on the distribution of charge on the shell.

- *Sample Problem 12-1* on page 365

Sample Problem 21.03 Charge sharing by two identical conducting spheres

In Fig. 21-9*a*, two identical, electrically isolated conducting spheres A and B are separated by a (center-to-center) distance a that is large compared to the spheres. Sphere A has a positive charge of $+Q$, and sphere B is electrically neutral. Initially, there is no electrostatic force between the spheres. (The large separation means there is no induced charge.)

(a) Suppose the spheres are connected for a moment by a conducting wire. The wire is thin enough so that any net charge on it is negligible. What is the electrostatic force between the spheres after the wire is removed?

KEY IDEAS

(1) Because the spheres are identical, connecting them means that they end up with identical charges (same sign and same amount). (2) The initial sum of the charges (including the signs of the charges) must equal the final sum of the charges.

Reasoning: When the spheres are wired together, the (negative) conduction electrons on B, which repel one another, have a way to move away from one another (along the wire to positively charged A, which attracts them—Fig. 21-9*b*). As B loses negative charge, it becomes positively charged, and as A gains negative charge, it becomes *less* positively charged. The transfer of charge stops when the charge on B has increased to $+Q/2$ and the charge on A has decreased to $+Q/2$, which occurs when $-Q/2$ has shifted from B to A.

After the wire has been removed (Fig. 21-9*c*), we can assume that the charge on either sphere does not disturb the uniformity of the charge distribution on the other sphere, because the spheres are small relative to their separation. Thus, we can apply the first shell theorem to each sphere. By Eq. 21-4 with $q_1 = q_2 = Q/2$ and $r = a$,

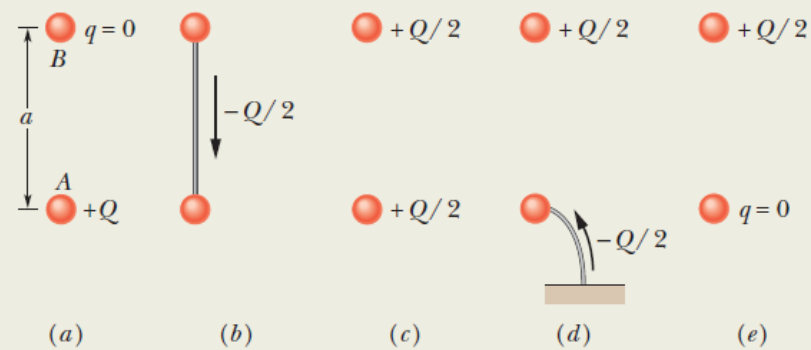


Figure 21-9 Two small conducting spheres A and B. (*a*) To start, sphere A is charged positively. (*b*) Negative charge is transferred from B to A through a connecting wire. (*c*) Both spheres are then charged positively. (*d*) Negative charge is transferred through a grounding wire to sphere A. (*e*) Sphere A is then neutral.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(Q/2)(Q/2)}{a^2} = \frac{1}{16\pi\varepsilon_0}\left(\frac{Q}{a}\right)^2. \qquad \text{(Answer)}$$

The spheres, now positively charged, repel each other.

(b) Next, suppose sphere A is grounded momentarily, and then the ground connection is removed. What now is the electrostatic force between the spheres?

Reasoning: When we provide a conducting path between a charged object and the ground (which is a huge conductor), we neutralize the object. Were sphere A negatively charged, the mutual repulsion between the excess electrons would cause them to move from the sphere to the ground. However, because sphere A is positively charged, electrons with a total charge of $-Q/2$ move *from* the ground up onto the sphere (Fig. 21-9*d*), leaving the sphere with a charge of 0 (Fig. 21-9*e*). Thus, the electrostatic force is again zero.

12-1~12-2 Electric Charge

Charge Is Quantized

- In Benjamin Franklin's day, electric charge was thought to be a continuous fluid—an idea that was useful for many purposes.
- However, we now know that fluids themselves, such as air and water, are not continuous but are made up of atoms and molecules; *matter is discrete*.
- Experiment shows that "electrical fluid" is also *not continuous but is made up of multiples of a certain elementary charge*. Any positive or negative charge q that can be detected can be written as
- $$q = ne, \qquad n = \pm 1, \pm 2, \pm 3, \ldots, \tag{12-7}$$

  in which e, the **elementary charge**, has the approximate value
- $$e = 1.602 \times 10^{-19}\ \mathrm{C}. \tag{12-8}$$
- The elementary charge e is *one of the important constants of nature*. The electron and proton both have a charge of magnitude e.
- When a physical quantity such as charge can have only discrete values rather than any value, we say that the quantity is **quantized**.

Particle	Symbol	Charge
Electron	e or e^-	$-e$
Proton	p	$+e$
Neutron	n	0

12-1~12-2 Electric Charge

Charge Is Conserved

- If you rub a glass rod with silk, a positive charge appears on the rod. Measurement shows that a negative charge of equal magnitude appears on the silk. This suggests that rubbing *does not create charge but only transfers it from one body to another*, upsetting the electrical neutrality of each body during the process.
- This hypothesis of **conservation of charge**, first put forward by Benjamin Franklin, has stood up under close examination, both for large-scale charged bodies and for atoms, nuclei, and elementary particles. *No exceptions have ever been found*.
- Thus, we add electric charge to our list of quantities—including energy and both linear momentum and angular momentum—that obey a **conservation law**.
- In applying the conservation-of-charge principle, we must add the charges *algebraically, with due regard for their signs*.

12-1~12-2 Electric Charge

Review & Summary

Electric Charge The strength of a particle's electrical interaction with objects around it depends on its **electric charge** (usually represented as q), which can be either positive or negative. Particles with the same sign of charge repel each other, and particles with opposite signs of charge attract each other. An object with equal amounts of the two kinds of charge is electrically neutral, whereas one with an imbalance is electrically charged and has an excess charge.

Conductors are materials in which a significant number of electrons are free to move. The charged particles in **nonconductors** (**insulators**) are not free to move.

Electric current i is the rate dq/dt at which charge passes a point:

$$i = \frac{dq}{dt} \quad \text{(electric current).}$$

Coulomb's Law Coulomb's law describes the electrostatic force (or electric force) between two charged particles. If the particles have charges q_1 and q_2, are separated by distance r, and are at rest (or moving only slowly) relative to each other, then the magnitude of the force acting on each due to the other is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1|\,|q_2|}{r^2} \quad \text{(Coulomb's law),}$$

where $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$ is the **permittivity constant**. The ratio $1/4\pi\varepsilon_0$ is often replaced with the **electrostatic constant** (or **Coulomb constant**) $k = 8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2}$.

The electrostatic force vector acting on a charged particle due to a second charged particle is either directly toward the second particle (opposite signs of charge) or directly away from it (same sign of charge). As with other types of forces, if multiple electrostatic forces act on a particle, the net force is the vector sum (not scalar sum) of the individual forces.

The two shell theorems for electrostatics are

Shell theorem 1: A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center.

Shell theorem 2: A charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell.

Charge on a conducting spherical shell spreads uniformly over the (external) surface.

The Elementary Charge Electric charge is quantized (restricted to certain values). The charge of a particle can be written as ne, where n is a positive or negative integer and e is the elementary charge, which is the magnitude of the charge of the electron and proton ($\approx 1.602 \times 10^{-19}$ C).

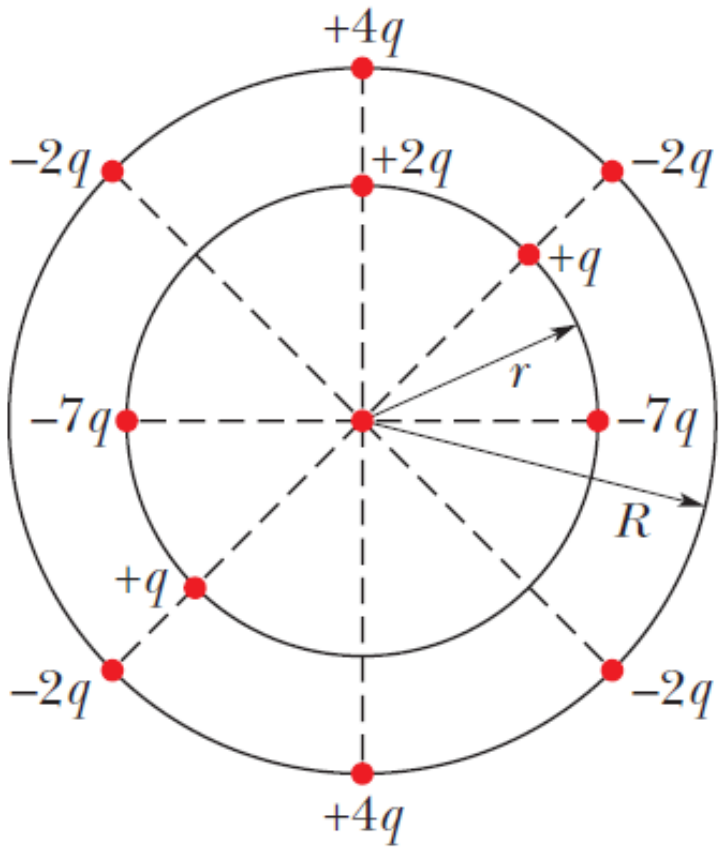
Conservation of Charge The net electric charge of any isolated system is always conserved.

- Please see the questions to answer on the following pages.

12-1~12-2 Electric Charge

Question-1 (page 396)

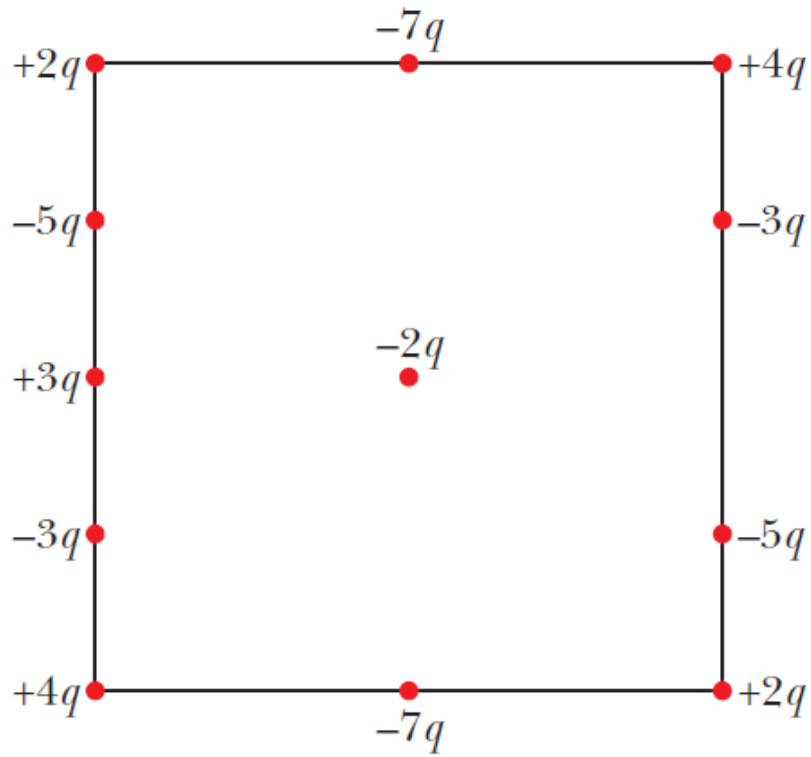
- In Figure below, a central particle of charge $-q$ is surrounded by two circular rings of charged particles. What are the magnitude and direction of the net electrostatic force on the central particle due to the other particles? (*Hint:* Consider symmetry.)



12-1~12-2 Electric Charge

Question-2 (page 396)

- In Figure below, a central particle of charge $-2q$ is surrounded by a square array of charged particles, separated by either distance d or $d/2$ along the perimeter of the square. What are the magnitude and direction of the net electrostatic force on the central particle due to the other particles? (*Hint:* Consideration of symmetry can greatly reduce the amount of work required here.)



12-3~12-5 Electric Field

- Figure right shows two positively charged particles. From the preceding discussions we know that an electrostatic force acts on particle 1 due to the presence of particle 2.
- We also know the force direction and, given some data, we can calculate the force magnitude.
- However, here is a leftover nagging question. How does particle 1 “know” of the presence of particle 2? That is, since the particles do not touch, how can particle 2 push on particle 1—how can there be such an *action at a distance*?
- One purpose of physics is to record observations about our world, such as the magnitude and direction of the push on particle 1.
- Another purpose is to provide an explanation of what is recorded.
- Our purpose here is to provide such an explanation to this nagging question about electric force at a distance.

12-3~12-5 Electric Field

- The explanation that we shall examine here is this: Particle 2 sets up an electric field at all points in the surrounding space, even if the space is a vacuum.
- If we place particle 1 at any point in that space, particle 1 knows of the presence of particle 2 because it is affected by the electric field particle 2 has already set up at that point.
- Thus, particle 2 pushes on particle 1 not by touching it as you would push on a coffee mug by making contact. Instead, particle 2 pushes by means of the electric field it has set up.
- Our goals here are to
- (1) define electric field,
- (2) discuss how to calculate it for various arrangements of charged particles and objects, and
- (3) discuss how an electric field can affect a charged particle (as in making it move).

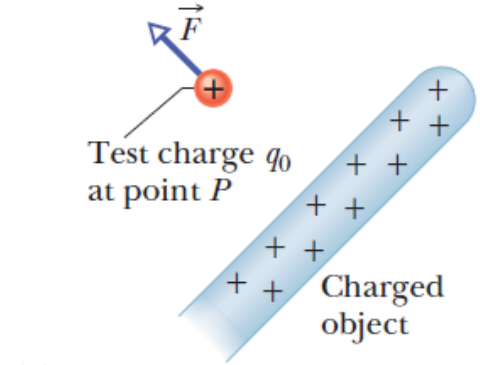
12-3~12-5 Electric Field

The Electric Field

- A lot of different fields are used in science and engineering.
- For example, a *temperature field* for an auditorium is the distribution of temperatures we would find by measuring the temperature at many points within the auditorium.
- Similarly, we could define a *pressure field* in a swimming pool. Such fields are examples of scalar fields because temperature and pressure are scalar quantities, having only magnitudes and not directions.
- In contrast, an ***electric field*** is a ***vector*** field because it is responsible for conveying the information for a force, i.e. ***electrostatic force*** , which involves both magnitude and direction.
- This field consists of *a distribution of electric field vectors* $\vec{E}$, one for each point in the space around a charged object.

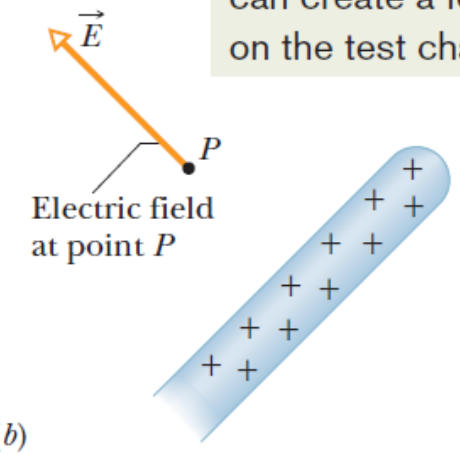
12-3~12-5 Electric Field

- In principle, we can define $\vec{E}$ at some point near the charged object, such as point P in Fig. (a), with this procedure:
- At P, we place a particle with a small positive charge q_0, called a test charge because we use it to test the field. (We want the charge to be small so that it does not disturb the object's charge distribution.)
- We then measure the electrostatic force that acts on the test charge.
- The electric field at that point is then
- $$\vec{E} = \frac{\vec{F}}{q_0} \quad \text{(electric field).} \qquad (12\text{-}12)$$
- Because the test charge is positive, the direction of $\vec{E}$ is the direction we measure for $\vec{F}$ .The magnitude of $\vec{E}$ at point P is F/q_0. As shown in Fig. (b), we always represent an electric field with an arrow with its tail anchored on the point where the measurement is made.



(a)

The rod sets up an electric field, which can create a force on the test charge.



(b)

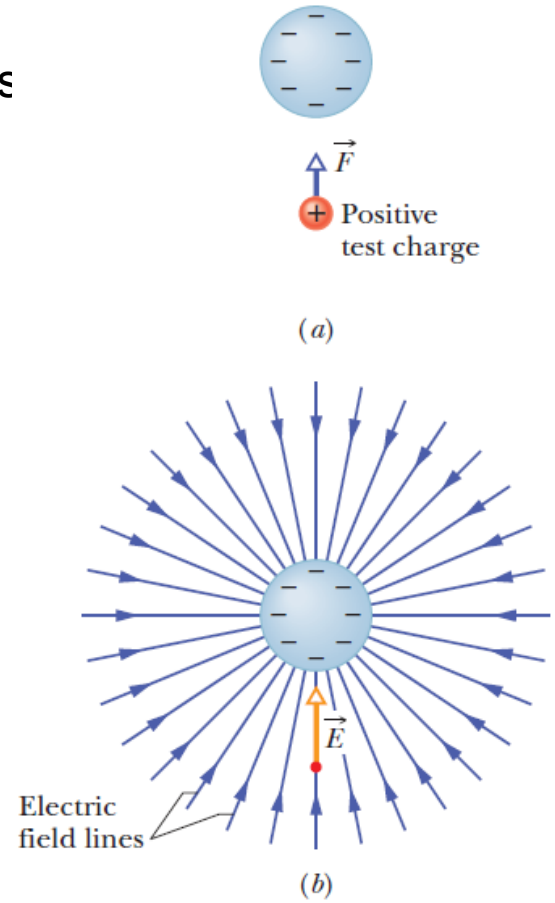
12-3~12-5 Electric Field

- From Eq. 12-12, we see that the SI unit for the electric field is the newton per coulomb (N/C).
- We can shift the test charge around to various other points, to measure the electric fields there, so that we can figure out the distribution of the electric field set up by the charged object.
- **Note:** *That field exists independent of the test charge.* It is something that a charged object sets up in the surrounding space (even vacuum), *independent of whether we happen to come along to measure it*.

12-3~12-5 Electric Field

Electric Field Lines--A Way of Visualizing Electric Fields

- Look at the space in the room around you. Can you visualize a field of vectors throughout that space—vectors with different magnitudes and directions?
- As impossible as that seems, *Michael Faraday* , who introduced the idea of electric fields in the 19th century, found a way.
- He envisioned lines, now called *electric field lines* , in the space around any given charged particle or object.
- Figure right gives an example in which a sphere is uniformly covered with negative charge. If we place a positive test charge at any point near the sphere (Fig. a), we find that an electrostatic force pulls on it toward the center of the sphere. Thus at every point around the sphere, *an electric field vector points radially inward toward the sphere*. We can represent this electric field with electric field lines as in Fig. b.



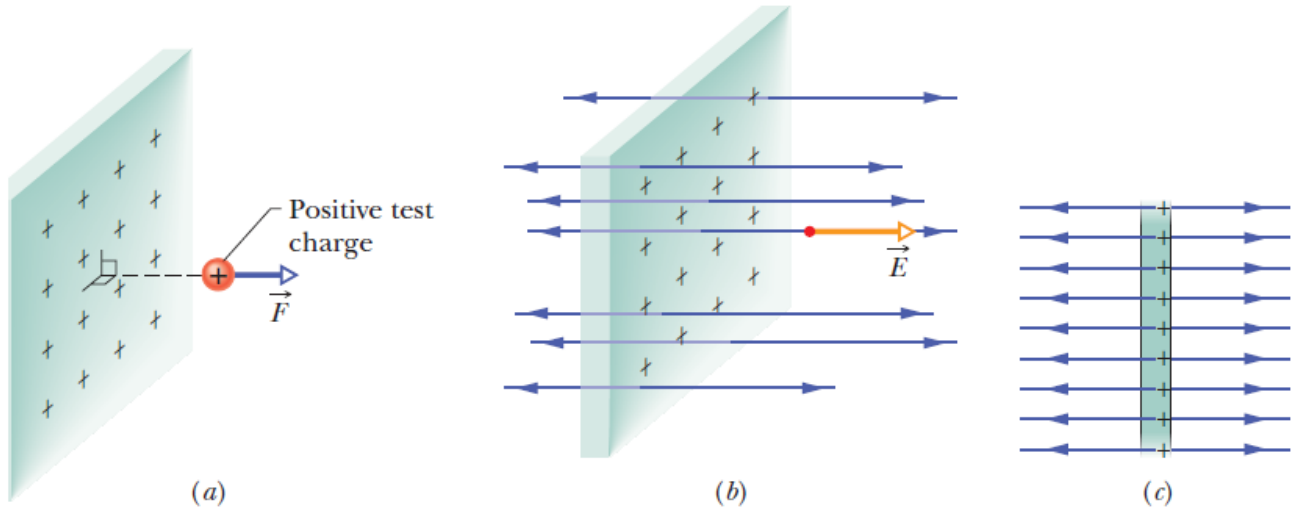
12-3~12-5 Electric Field

- The rules for drawing electric fields lines are these:
- (1) At any point, the electric field vector must be tangent to the electric field line through that point and in the same direction.
- (2) In a plane perpendicular to the field lines, the relative density of the lines represents the relative magnitude of the field there, with greater density for greater magnitude.
- If the sphere in the figure shown on the last page were uniformly covered with positive charge, the electric field vectors at all points around it would be radially outward and thus so would the electric field lines. So, we have the following rule:

> Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).

- In figure b on the last page, the electric field lines originate on distant positive charges that are not shown.

12-3~12-5 Electric Field



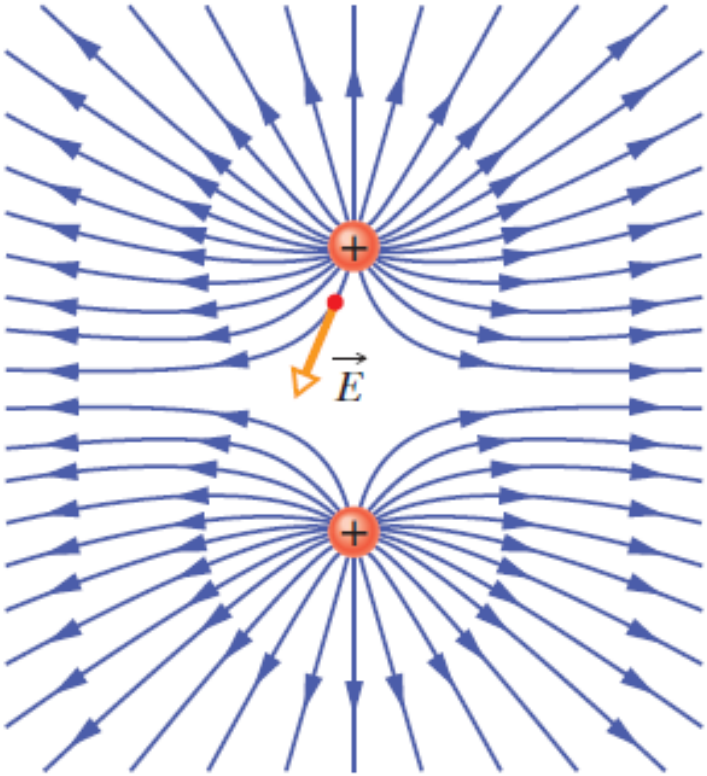
- For another example, Fig. a shows part of an infinitely large, nonconducting sheet (or plane) with ***a uniform distribution of positive charge on one side*** .
- If we place a positive test charge at any point near the sheet (on either side), we find that the electrostatic force on the particle is ***outward and perpendicular to the sheet*** . The perpendicular orientation is reasonable because any force component that is, say, upward is balanced out by an equal component that is downward. That leaves only outward, and thus the electric field vectors and the electric field lines must also be outward and perpendicular to the sheet, as shown in Figs. b and c.

12-3~12-5 Electric Field

- Because the charge on the sheet is uniform, the field vectors and the fieldnlines are also. Such a field is a ***uniform electric field*** , meaning that the electric field has the *same magnitude and direction at every point within the field*. (This is a lot easier to work with than a ***nonuniform field*** , where there is variation from point to point.)
- Of course, there is no such thing as an infinitely large sheet. That is just a way of saying that we are measuring the field at points close to the sheet relative to the size of the sheet and that we are not near an edge.

12-3~12-5 Electric Field

- Figure right shows the field lines for two particles with equal positive charges.
- Now the field lines are curved, but the rules still hold:
- (1) the electric field vector at any given point must be tangent to the field line at that point and in the same direction, as shown for one vector, and
- (2) a closer spacing means a larger field magnitude. To imagine the full three-dimensional pattern of field lines around the particles, mentally rotate the pattern in Figure right around the axis of symmetry, which is a vertical line through both particles.



12-3~12-5 Electric Field

The Electric Field Due to a Point Charge

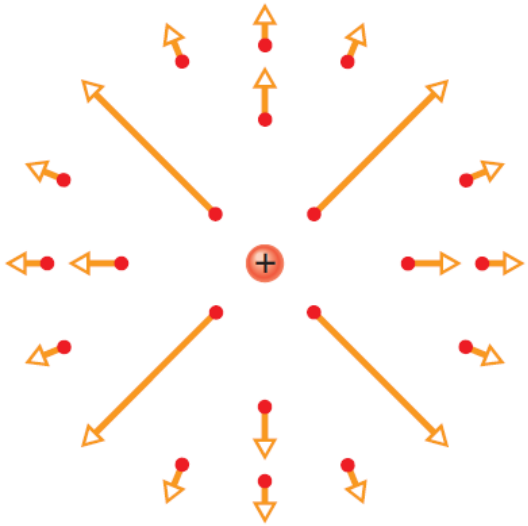
- To find the electric field due to a charged particle (often called a *point charge*), we place a positive test charge at any point near the particle, at distance r . From Coulomb's law, the force on the test charge due to the particle with charge q is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{\mathrm{r}}.$$

- As previously, the direction of $\vec{F}$ is directly away from the particle if q is positive (because q_0 is positive) and directly toward it if q is negative.
- From Eq. 12-12, we can now write the electric field set up by the particle (at the location of the test charge) as
- $$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathrm{r}} \quad \text{(charged particle).}$$
- Let's think through the directions again. *The direction of $\vec{E}$ matches that of the force on the positive test charge* : directly away from the point charge if q is positive and directly toward it if q is negative.

12-3~12-5 Electric Field

- We can find the magnitude at any given distance r by converting the equation for $\vec{E}$ to a magnitude form:
- $$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2} \quad \text{(charged particle).} \qquad (12\text{-}14)$$
- Eq. 12-14 gives magnitude E only. We must think about the direction separately.
- Figure below gives a number of electric field vectors at points around a positively charged particle, but be careful. Each vector represents the vector quantity at the point where the tail of the arrow is anchored. The vector is not something that stretches from a “here” to a “there” as with a displacement vector.



12-3~12-5 Electric Field

- In general, if several electric fields are set up at a given point by several charged particles, we can find the net field by placing a positive test particle at the point and then writing out the force acting on it due to each particle, such as $\vec{F}_0$ due to particle 1. Forces obey the principle of superposition, so we just add the forces as vectors:

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}.$$

- To change over to electric field, we repeatedly use Eq. 12-12 for each of the individual forces:

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0}$$

- $$= \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n. \qquad (12\text{-}15)$$

- This tells us that electric fields also obey the principle of superposition. If you want the net electric field at a given point due to several particles, find the electric field due to each particle and then sum the fields as vectors.
- See *Sample Problem 12-2* on page.

12-3~12-5 Electric Field

- *Sample Problem 12-2* on page 370

Sample Problem 22.01 Net electric field due to three charged particles

Figure 22-7*a* shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field $\vec{E}$ is produced at the origin?

KEY IDEA

Charges q_1, q_2, and q_3 produce electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, respectively, at the origin, and the net electric field is the vector sum $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$. To find this sum, we first must find the magnitudes and orientations of the three field vectors.

Magnitudes and directions: To find the magnitude of $\vec{E}_1$, which is due to q_1, we use Eq. 22-3, substituting d for r and $2Q$ for q and obtaining

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}.$$

Similarly, we find the magnitudes of $\vec{E}_2$ and $\vec{E}_3$ to be

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} \quad \text{and} \quad E_3 = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}.$$

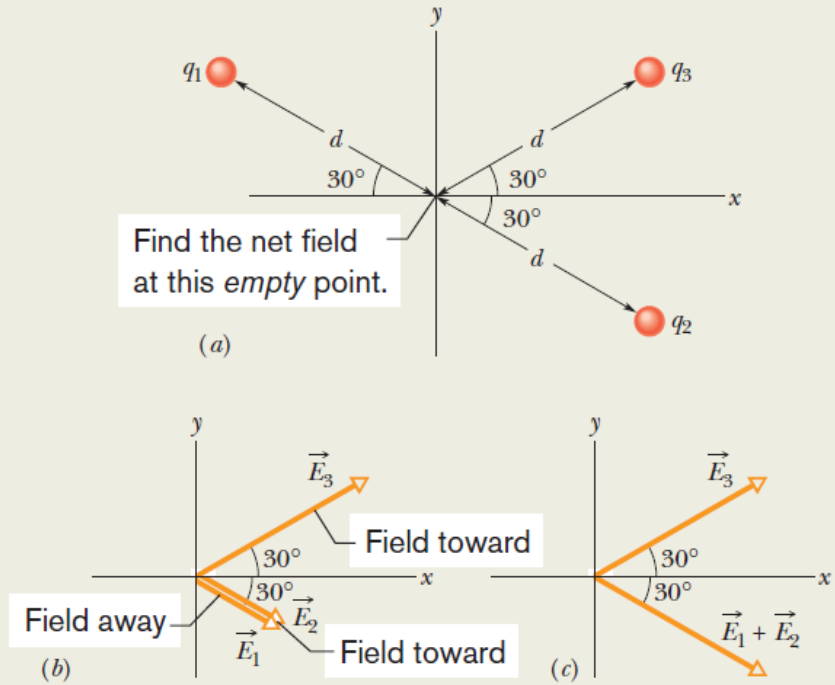


Figure 22-7 (*a*) Three particles with charges q_1, q_2, and q_3 are at the same distance d from the origin. (*b*) The electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, at the origin due to the three particles. (*c*) The electric field vector $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$ at the origin.

12-3~12-5 Electric Field

- ***Sample Problem 12-2*** on page 370

We next must find the orientations of the three electric field vectors at the origin. Because q_1 is a positive charge, the field vector it produces points directly *away* from it, and because q_2 and q_3 are both negative, the field vectors they produce point directly *toward* each of them. Thus, the three electric fields produced at the origin by the three charged particles are oriented as in Fig. 22-7*b*. (*Caution:* Note that we have placed the tails of the vectors at the point where the fields are to be evaluated; doing so decreases the chance of error. Error becomes very probable if the tails of the field vectors are placed on the particles creating the fields.)

Adding the fields: We can now add the fields vectorially just as we added force vectors in Chapter 21. However, here we can use symmetry to simplify the procedure. From Fig. 22-7*b*, we see that electric fields $\vec{E}_1$ and $\vec{E}_2$ have the same direction. Hence, their vector sum has that direction and has the magnitude

$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2},$$

which happens to equal the magnitude of field $\vec{E}_3$.

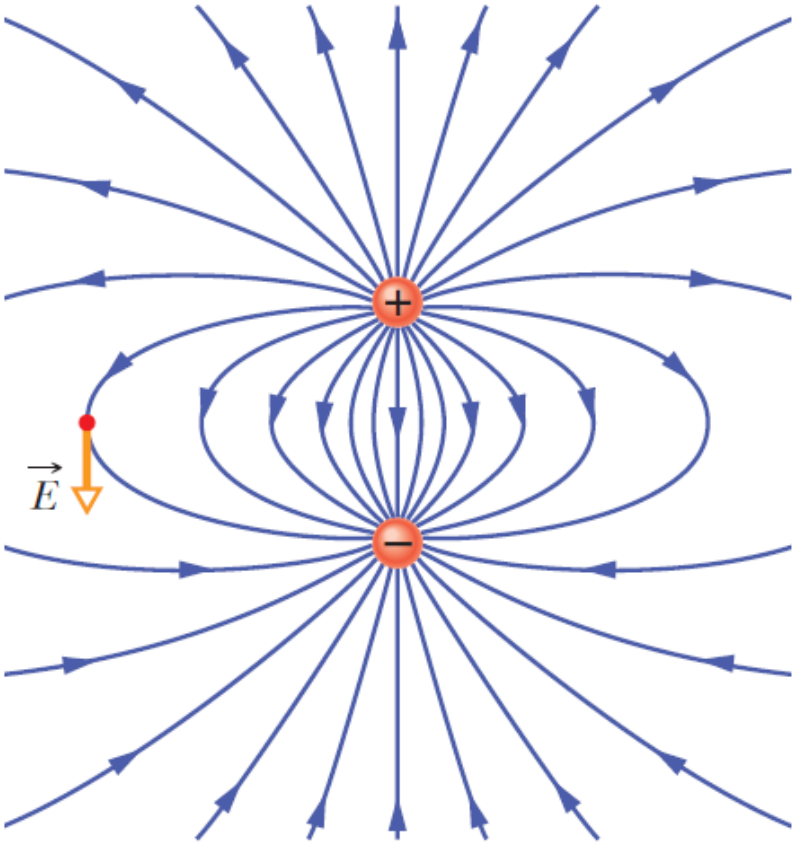
We must now combine two vectors, $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$, that have the same magnitude and that are oriented symmetrically about the x axis, as shown in Fig. 22-7*c*. From the symmetry of Fig. 22-7*c*, we realize that the equal y components of our two vectors cancel (one is upward and the other is downward) and the equal x components add (both are rightward). Thus, the net electric field $\vec{E}$ at the origin is in the positive direction of the x axis and has the magnitude

$$E = 2E_{3x} = 2E_3 \cos 30^\circ = (2)\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}(0.866) = \frac{6.93Q}{4\pi\varepsilon_0 d^2}. \qquad \text{(Answer)}$$

12-3~12-5 Electric Field

The Electric Field Due to an Electric Dipole

- Figure right shows the pattern of electric field lines for two particles that have the same charge magnitude q but opposite signs, a very common and important arrangement known as an ***electric dipole***. The particles are separated by distance d and lie along the *dipole axis*, an axis of symmetry around which you can imagine rotating the pattern in the Figure.
- Let's label that axis as a z axis. Here we restrict our interest to the magnitude and direction of the electric field at an arbitrary point P along the dipole axis, at distance z from the dipole's midpoint.



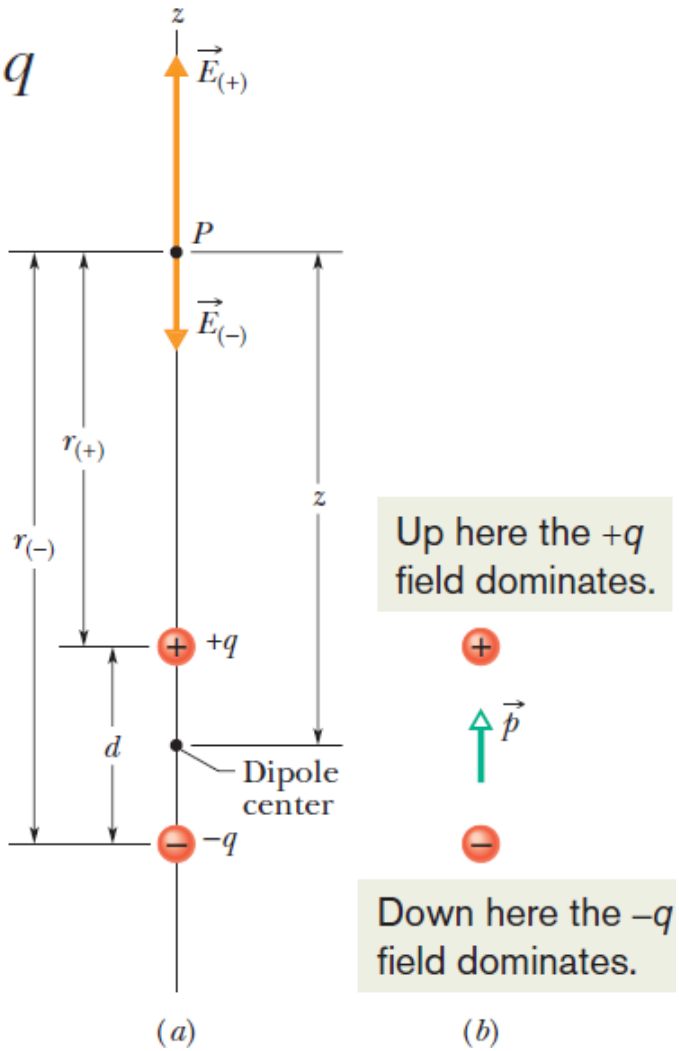
12-3~12-5 Electric Field

- Figure 12-12a shows the electric fields set up at P by each particle. The nearer particle with charge $+q$ sets up field $E_{(+)}$ in the positive direction of the z axis (directly away from the particle). The farther particle with charge $-q$ sets up a smaller field $E_{(-)}$) in the negative direction (directly toward the particle).
- We want the net field at P, as given by Eq. 12-15. However, because the field vectors are along the same axis, let's simply indicate the vector directions with plus and minus signs. Then we can write the magnitude of the net field at P as

$$E = E_{(+)} - E_{(-)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2}$$

- $$= \frac{q}{4\pi\varepsilon_0(z - \frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z + \frac{1}{2}d)^2}. \quad (12\text{-}16)$$



12-3~12-5 Electric Field

- After a little algebra, we can rewrite this equation as
- $$E = \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{1}{\left(1-\frac{d}{2z}\right)^2} - \frac{1}{\left(1+\frac{d}{2z}\right)^2}\right). \qquad (12\text{-}17)$$
- After forming a common denominator and multiplying its terms, we come to

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\frac{2d/z}{\left(1-\left(\frac{d}{2z}\right)^2\right)^2} = \frac{q}{2\pi\varepsilon_0 z^3}\frac{d}{\left(1-\left(\frac{d}{2z}\right)^2\right)^2}.$$

- We are usually interested in the electrical effect of a dipole only at distances that are large compared with the dimensions of the dipole—that is, at distances such that $z \gg d$ .At such large distances, we have $d/2z \ll 1$. Thus, for approximation, we can neglect the $d/2z$ term in the denominator, which leaves us with
- $$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}. \qquad (12\text{-}19)$$

12-3~12-5 Electric Field

- The product qd , which involves the two intrinsic properties q and d of the dipole, is the magnitude p of a vector quantity known as the ***electric dipole moment*** $\vec{p}$ of the dipole. (The unit of $\vec{p}$ is the coulomb-meter.) Thus, we can write Eq. 12-19 as
- $$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \quad \text{(electric dipole).} \qquad (12\text{-}20)$$
- The direction of $\vec{p}$ is taken to be from the negative to the positive end of the dipole, as indicated in Fig. 12-12b. We can use the direction of $\vec{p}$ to specify the orientation of a dipole.
- Equation 12-20 shows that, if we measure the electric field of a dipole only at distant points, we can never find q and d separately; instead, we can find only their product.
- Although Eq. 12-20 holds only for distant points along the dipole axis, it turns out that E for a dipole varies as $1/r^3$ for ***all*** distant points, regardless of whether they lie on the dipole axis; here r is the distance between the point in question and the dipole center.

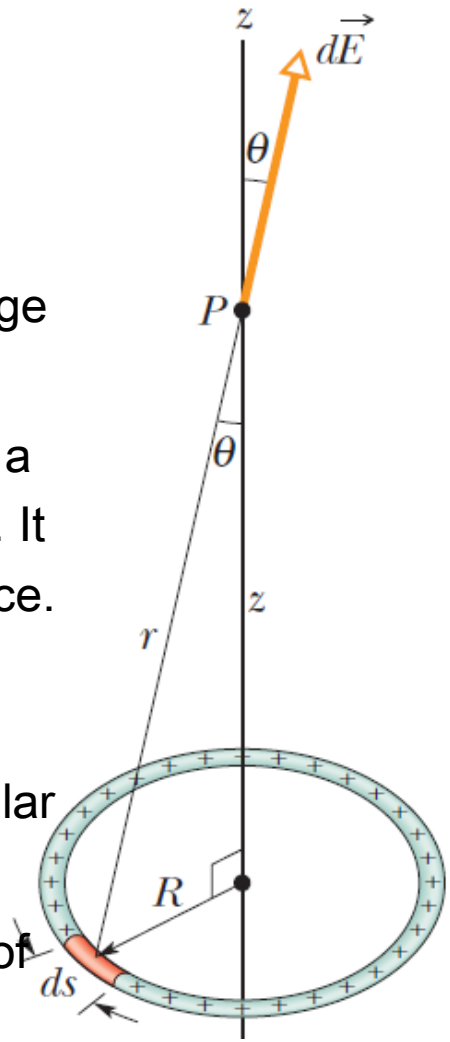
12-3~12-5 Electric Field

- Inspection of Fig. 12-12 and of the field lines shows that the direction of $\vec{E}$ for distant points on the dipole axis is always the direction of the dipole moment vector $\vec{p}$ .This is true whether point P in Fig. 12-12a is on the upper or the lower part of the dipole axis.
- Inspection of Eq. 12-20 shows that if you double the distance of a point from a
- dipole, the electric field at the point drops by a factor of 8. If you double the distance from a single point charge, however, the electric field drops only by a factor of 4. Thus the electric field of a dipole *decreases more rapidly with distance* than does the electric field of a single charge.
- The physical reason for this rapid decrease in electric field for a dipole is that from distant points a dipole looks like two particles that almost—but not quite—coincide. Thus, because they have charges of equal magnitude but opposite signs, *their electric fields at distant points almost—but not quite—cancel each other*.

12-3~12-5 Electric Field

The Electric Field Due to a Line of Charge

- So far we have dealt with only charged particles, a single particle or a simple collection of them. We now turn to a much more challenging situation in which a thin (approximately one-dimensional) object such as a rod or ring is charged with a huge number of particles, more than we could ever even count.
- Figure 12-14 (see the right) shows a thin ring of radius R with a uniform distribution of positive charge along its circumference. It is made of plastic, which means that the charge is fixed in place. The ring is surrounded by a pattern of electric field lines, but here we restrict our interest to an arbitrary point P on the central axis (the axis through the ring's center and perpendicular to the plane of the ring), at distance z from the center point.
- The charge of an extended object is often conveyed in terms of a charge density rather than the total charge. For a line of charge, we use the *linear charge density* λ (the charge per unit length), with the SI unit of coulomb per meter.



12-3~12-5 Electric Field

Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m^2
Volume charge density	ρ	C/m^3

- Table 12-2 (the table above) shows the other charge densities that we shall be using for charged surfaces and volumes.
- ***First Big Problem.*** So far, we have an equation for the electric field of a particle. Now take a look at the ring in Fig. 12-14.That clearly is not a particle and so Eq. 12-14 does not apply. So what do we do?
- The answer is to mentally divide the ring into differential elements of charge that are so small that we can treat them as though they are particles. Then we can apply Eq. 12-14.

12-3~12-5 Electric Field

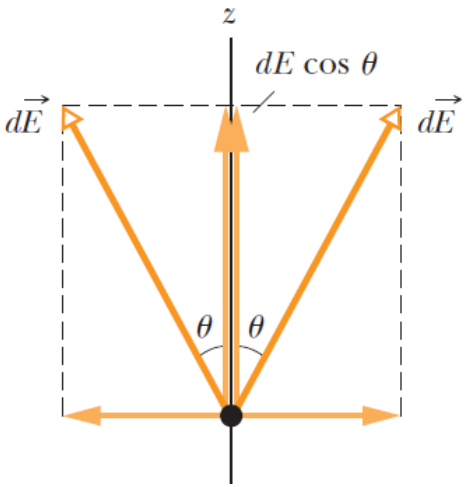
- ***Second Big Problem.*** We now know to apply Eq. 12-14 to each charge element dq and can write an expression for its contribution of electric field $d\vec{E}$. However, each such contributed field vector at P is in its own direction. How can we add them to get the net field at P?
- The answer is to split the vectors into components and then separately sum one set of components and then the other set. However, first we check to see if one set simply all cancels out. (Canceling out components saves lots of work.)
- ***Third Big Problem.*** There is a huge number of dq elements in the ring and thus a huge number of components to add up, even if we can cancel out one set of components. How can we add up more components than we could even count?
- The answer is to add them by means of integration.
- ***Do It.*** Let's do all this (but again, be aware of the general procedure, not just the fine details).We arbitrarily pick the charge element shown in Fig. 12-14. Let ds be the arc length of that (or any other) dq element. Then in terms of the linear density λ (the charge per unit length), we have
- $$dq = \lambda\, ds. \qquad (12\text{-}21)$$

12-3~12-5 Electric Field

- *An Element's Field.* This charge element sets up the differential electric field $d\vec{E}$ at P , at distance r from the element, as shown in Fig. 12-14. Next we rewrite the field equation for a particle (Eq. 12-14) in terms of our new symbols dE and dq , but then we replace dq using Eq. 12-21.The field magnitude due to the charge element is
- $$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}. \qquad (12\text{-}22)$$
- Notice that the symbol r is the hypotenuse of the right triangle displayed in Fig. 12-14.Thus, we can replace r by rewriting Eq. 12-22 as
- $$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2 + R^2)}. \qquad (12\text{-}23)$$
- Because every charge element has the same charge and the same distance
- from point P , Eq. 12-23 gives the field magnitude contributed by each of them.
- Figure 12-14 also tells us that each contributed leans at angle θ to the central axis (the z axis) and thus has components perpendicular and parallel to that axis.

12-3~12-5 Electric Field

- ***Canceling Components.*** In Fig. 12-14, consider the charge element on the opposite side of the ring. It too contributes the field magnitude dE but the field vector leans at angle θ in the opposite direction from the vector from our first charge element, as indicated in the side view of figure right.



- Thus the two perpendicular components cancel. All around the ring, this cancelation occurs for every charge element and its *symmetric partner* on the opposite side of the ring. So we can neglect all the perpendicular components.
- ***Adding Components.*** All the remaining parallel components are in the positive direction of the z axis, so we can just add them up as scalars.
- From figure right, we see that the parallel components each have magnitude $dE \cos \theta$,
- $$\cos \theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}}. \qquad (12\text{-}24)$$

12-3~12-5 Electric Field

- Multiplying Eq. 12-23 by Eq. 12-24 gives us the parallel field component from each charge element:
- $$dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{z\lambda}{(z^2+R^2)^{3/2}}\,ds. \qquad (12\text{-}25)$$
- ***Integrating.*** Because we must sum a huge number of these components, each small, we set up an integral that moves along the ring, from element to element, from a starting point (call it $s = 0$) through the full circumference ($s = 2\pi R$). Only the quantity s varies as we go through the elements; the other symbols in Eq. 12-25 remain the same, so we move them outside the integral. We find,
- $$E = \int dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}\int_0^{2\pi R} ds$$
  $$= \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}. \qquad (12\text{-}24)$$
- This is a fine answer, but we can also switch to the total charge by $\lambda = q/(2\pi R)$:

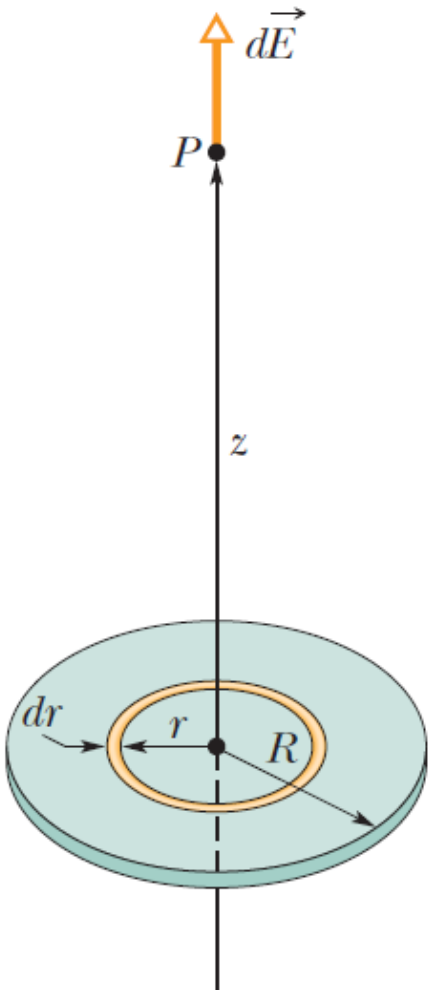
12-3~12-5 Electric Field

- $$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \quad \text{(charged ring).} \qquad (12\text{-}27)$$
- If the charge on the ring is negative, instead of positive as we have assumed, the magnitude of the field at P is still given by Eq. 12-27. However, the electric field vector then points toward the ring instead of away from it.
- Let us check Eq. 12-27 for a point on the central axis that is so far away that $z \gg R$. For such a point, the expression $z^2 + R^2$ can be approximated as z^2 , and Eq. 12-27 becomes
- $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2} \quad \text{(charged ring at large distance).} \qquad (12\text{-}28)$$
- This is a reasonable result because from a large distance, the ring “looks like” a point charge. If we replace z with r in Eq. 12-28, we indeed do have the magnitude of the electric field due to a point charge, as given by Eq. 12-14.
- Let us next check Eq. 12-27 for a point at the center of the ring—that is, for $z = 0$. At that point, Eq. 12-27 tells us that $E = 0$.

12-3~12-5 Electric Field

The Electric Field Due to a Charged Disk

- Now we switch from a line of charge to a surface of charge by examining the electric field of a circular plastic disk, with a radius R and a uniform surface charge density σ (charge per unit area, Table 12-2) on its top surface.
- The disk sets up a pattern of electric field lines around it, but here we restrict our attention to the electric field at an arbitrary point P on the central axis, at distance z from the center of the disk, as indicated in Fig. 12-16 (see the right figure).
- We could proceed as in the preceding module (**The Electric Field Due to a Line of Charge**) but set up a two-dimensional integral to include all of the field contributions from the two-dimensional distribution of charge on the top surface.
- However, we can save a lot of work with a neat shortcut using our earlier work with the field on the central axis of a thin ring.



12-3~12-5 Electric Field

- We superimpose a ring on the disk as shown in Fig. 12-16, at an arbitrary radius of $r \leq R$. The ring is so thin that we can treat the charge on it as a charge element dq. To find its small contribution dE to the electric field at point P , we rewrite Eq. 12-27 in terms of the ring's charge dq and radius r :

$$dE = \frac{dq\, z}{4\pi\varepsilon_0 (z^2 + r^2)^{3/2}}.$$

- The ring's field points in the positive direction of the z axis.
- To find the total field at P , we are going to integrate from the center of the disk at $r = 0$ out to the rim at $r = R$ so that we sum all the dE contributions (by sweeping our arbitrary ring over the entire disk surface). That means we have to integrate with respect to a variable radius r of the ring.
- To do that, we get dr into the expression by substituting for dq . Because the ring is so thin, call its thickness dr . Then its surface area dA is the product of its circumference $2\pi r$ and thickness dr. So, in terms of the surface charge density σ , we have
- $$dq = \sigma\, dA = \sigma (2\pi r\, dr). \qquad (12\text{-}33)$$

12-3~12-5 Electric Field

- Thus, After substituting (12-33) into dE , we can sum all the dE contributions with
- $$E = \int dE = \frac{\sigma z}{4\varepsilon_0} \int_0^R (z^2 + r^2)^{-3/2}(2r)\, dr, \qquad (12\text{-}35)$$
- To solve this integral, we cast it in the form $\int X^m\, dX$ by setting $X = (z^2 + r^2)$, $m = -\frac{3}{2}$, and $dX = (2r)\, dr$. For the recast integral we have

$$\int X^m\, dX = \frac{X^{m+1}}{m+1},$$

and so Eq. 12-35 becomes

$$E = \frac{\sigma z}{4\varepsilon_0} \left[\frac{(z^2 + r^2)^{-1/2}}{-\frac{1}{2}} \right]_0^R. \qquad (12\text{-}36)$$

- Taking the limits in Eq. 12-36 and rearranging, we find
- $$E = \frac{\sigma}{2\varepsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right) \quad \text{(charged disk)} \qquad (12\text{-}37)$$

12-3~12-5 Electric Field

- If we let $R \longrightarrow \infty$ while keeping z finite, the second term in the parentheses in Eq. 12-37 approaches zero, and this equation reduces to

$$E = \frac{\sigma}{2\varepsilon_0} \quad \text{(infinite sheet).} \tag{12-38}$$

- This is the electric field produced by an infinite sheet of uniform charge located on one side of a nonconductor such as plastic. The electric field lines for such a situation are shown in Fig. 12-7.
- We also get Eq. 12-38 if we let $z \longrightarrow 0$ in Eq. 12-37 while keeping R finite. This shows that at points very close to the disk, the electric field set up by the disk is the same as if the disk were infinite in extent.

12-3~12-5 Electric Field

A Point Charge in an Electric Field

- In the preceding four modules we worked at the first of our two tasks: given a charge distribution, to find the electric field it produces in the surrounding space.
- Here we begin the second task: to ***determine what happens to a charged particle when it is in an electric field*** set up by other stationary or slowly moving charges.
- What happens is that *an electrostatic force acts on the particle,* as given by
- $$\vec{F} = q\vec{E}, \qquad (12\text{-}39)$$
- It tells us

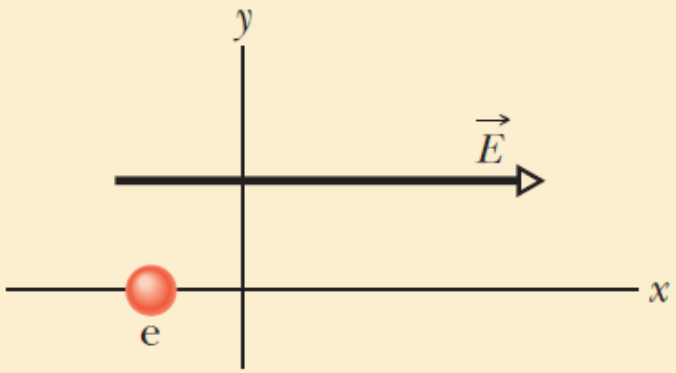
> The electrostatic force $\vec{F}$ acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge q of the particle is positive and has the opposite direction if q is negative.

- See ***CHECKPOINT 2*** on page 378 of the textbook. Please also read the following materials about "Measuring the Elementary Charge," "Ink-Jet Printing," and "Electrical Breakdown and Sparking" on pages 378-379 of the textbook.

12-3~12-5 Electric Field

- *CHECKPOINT 2* on page 378 of the textbook

(a) In the figure, what is the direction of the electrostatic force on the electron due to the external electric field shown? (b) In which direction will the electron accelerate if it is moving parallel to the y axis before it encounters the external field? (c) If, instead, the electron is initially moving rightward, will its speed increase, decrease, or remain constant?



12-3~12-5 Electric Field

Supplement-1: The Vector Product

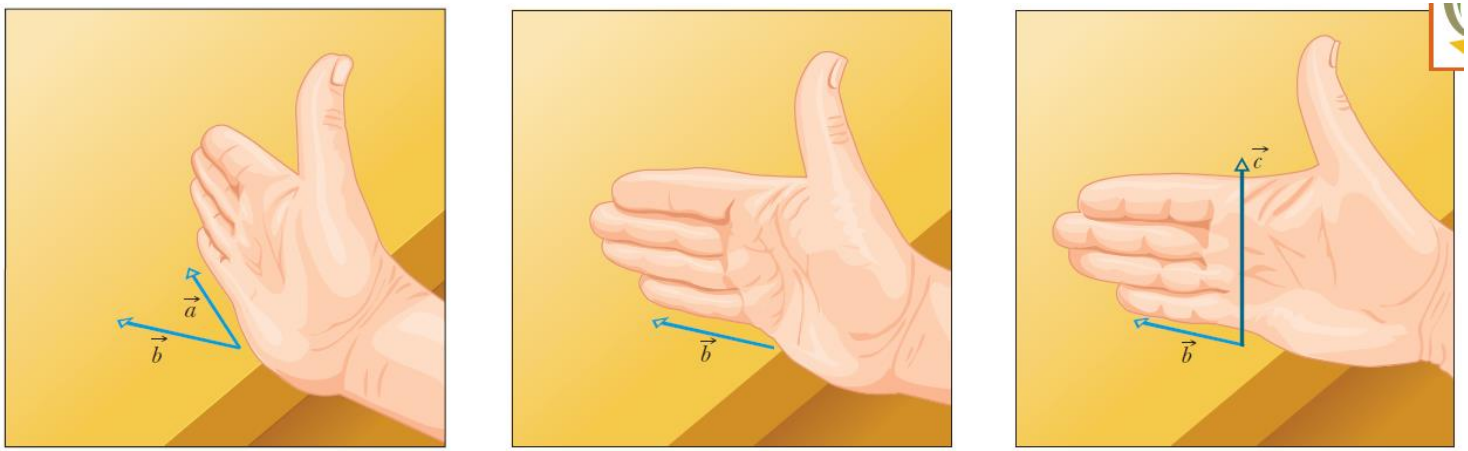
- The vector product of $\vec{a}$ and $\vec{b}$, written $\vec{a} \times \vec{b}$, produces a third vector whose magnitude is
- $$c = ab \sin \phi \qquad (12\text{-}39)$$
- where ϕ is the *smaller* of the two angles between $\vec{a}$ and $\vec{b}$. (You must use the smaller of the two angles between the vectors because $\sin \phi$ and $\sin(360° - \phi)$ differ in algebraic sign.
- Because of the notation, $\vec{a} \times \vec{b}$ is also known as the ***cross product***, and in speech it is "a cross b."

If $\vec{a}$ and $\vec{b}$ are parallel or antiparallel, $\vec{a} \times \vec{b} = 0$. The magnitude of $\vec{a} \times \vec{b}$, which can be written as $|\vec{a} \times \vec{b}|$, is maximum when $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

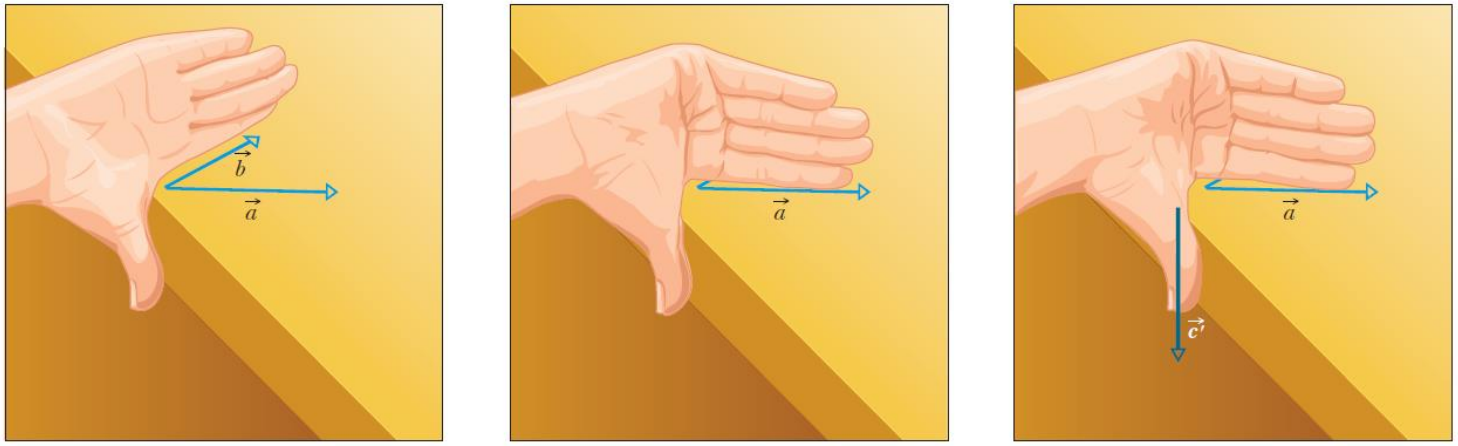
- The direction of $\vec{c}$ is perpendicular to the plane that contains $\vec{a}$ and $\vec{b}$.
- Figure a on next page shows how to determine the direction of $\vec{c}$ with what is known as a ***right-hand rule***.

12-3~12-5 Electric Field

- (a) Sweep vector $\vec{a}$ into vector $\vec{b}$ with the fingers of your right hand. Your outstretched thumb shows the direction of vector $\vec{c} = \vec{a} \times \vec{b}$.
- (b) Showing that $\vec{b} \times \vec{a}$ is the reverse of $\vec{a} \times \vec{b}$.



(a)



(b)

12-3~12-5 Electric Field

- **Right-hand rule:** Place the vectors $\vec{a}$ and $\vec{b}$ tail to tail without altering their orientations, and imagine a line that is perpendicular to their plane where they meet. Pretend to place your right hand around that line in such a way that your fingers would sweep $\vec{a}$ into $\vec{b}$ through the smaller angle between them. Your outstretched thumb points in the direction of $\vec{c}$.
- **Note:** the commutative law does not apply to a vector product, because

$$\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b}).$$

- In unit-vector notation,

$$\vec{a} \times \vec{b} = (a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}) \times (b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} + b_z\hat{\mathrm{k}})$$

$$\hat{\mathrm{i}} \cdot \hat{\mathrm{i}} = \hat{\mathrm{j}} \cdot \hat{\mathrm{j}} = \hat{\mathrm{k}} \cdot \hat{\mathrm{k}} = 1, \quad \hat{\mathrm{i}} \cdot \hat{\mathrm{j}} = \hat{\mathrm{j}} \cdot \hat{\mathrm{k}} = \hat{\mathrm{k}} \cdot \hat{\mathrm{i}} = 0,$$

$$\hat{\mathrm{i}} \times \hat{\mathrm{i}} = \hat{\mathrm{j}} \times \hat{\mathrm{j}} = \hat{\mathrm{k}} \times \hat{\mathrm{k}} = 0,$$

$$\hat{\mathrm{i}} \times \hat{\mathrm{j}} = \hat{\mathrm{k}}, \quad \hat{\mathrm{j}} \times \hat{\mathrm{k}} = \hat{\mathrm{i}}, \quad \hat{\mathrm{k}} \times \hat{\mathrm{i}} = \hat{\mathrm{j}}$$

12-3~12-5 Electric Field

- Other properties of cross product:

$$\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$$

$$(s\vec{a}) \times \vec{b} = \vec{a} \times (s\vec{b}) = s(\vec{a} \times \vec{b}) \qquad (s = \text{a scalar})$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b})$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a} = \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$= \hat{\mathrm{i}} \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{\mathrm{j}} \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{\mathrm{k}} \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

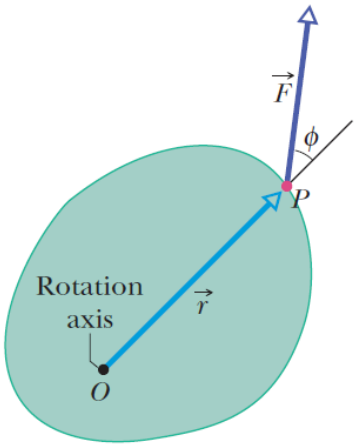
$$= (a_y b_z - b_y a_z)\hat{\mathrm{i}} + (a_z b_x - b_z a_x)\hat{\mathrm{j}}$$

$$+ (a_x b_y - b_x a_y)\hat{\mathrm{k}}$$

12-3~12-5 Electric Field

Supplement-2: Torque

- If you want to open a heavy door, you must certainly apply a force, but that is not enough. Where you apply that force and in what direction you push are also important.
- If you apply your force nearer to the hinge line than the knob, or at any angle other than 90° to the plane of the door, you must use a greater force than if you apply the force at the knob and perpendicular to the door's plane. That is why a doorknob is always located as far as possible from the door's hinge line.
- Figure a shows a cross section of a body that is free to rotate about an axis passing through O and perpendicular to the cross section.
- A force $\vec{F}$ is applied at point P, whose position relative to O is defined by a position vector $\vec{r}$.
- The directions of vectors $\vec{F}$ and $\vec{r}$ make an angle ϕ with each other.



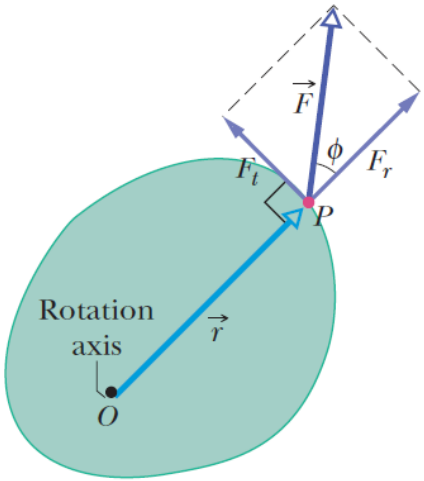
(a)

The torque due to this force causes rotation around this axis (which extends out toward you).

12-3~12-5 Electric Field

- To determine how $\vec{F}$ results in a rotation of the body around the rotation axis, we resolve $\vec{F}$ into two components (Fig. b).
- One component, called the ***radial component*** F_r, points along $\vec{r}$. This component ***does not*** cause rotation, because it acts along a line that extends through O. (If you pull on a door parallel to the plane of the door, you do not rotate the door.)
- The other component of $\vec{F}$, called the ***tangential component*** F_t, is perpendicular to $\vec{r}$ and has magnitude $F_t = F\sin\phi$. This component ***does*** cause rotation.
- The ability of $\vec{F}$ to rotate the body depends not only on the magnitude of its tangential component F_t, but also on just how far from O the force is applied. To include both these factors, we define a quantity called ***torque*** τ as the product of the two factors and write it as

$$\tau = (r)(F\sin\phi).$$



(b)

But actually only the *tangential* component of the force causes the rotation.

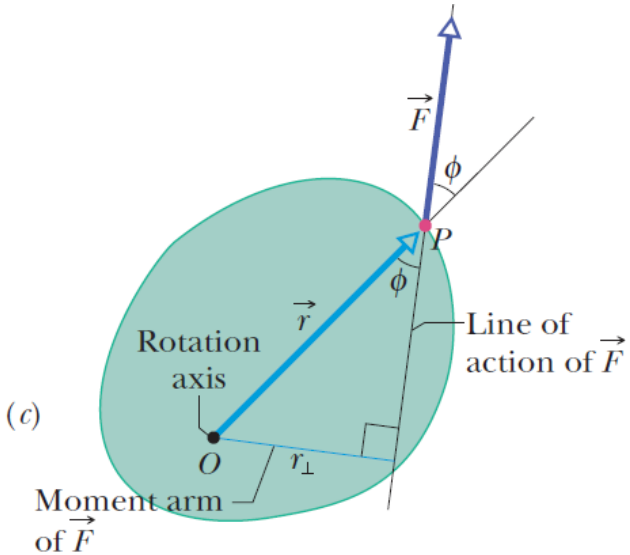
12-3~12-5 Electric Field

- Two equivalent ways of computing the torque are

$$\tau = (r)(F\sin\phi) = rF_t$$

$$\tau = (r\sin\phi)(F) = r_\perp F,$$

where $r_\perp$ is the perpendicular distance between the rotation axis at O and an extended line running through the vector $\vec{F}$ (Fig. c).This extended line is called the *line of action* of $\vec{F}$, and $r_\perp$ is called the *moment arm* of $\vec{F}$.



You calculate the same torque by using this moment arm distance and the full force magnitude.

- Torque, which comes from the Latin word meaning “to twist,” may be loosely identified as the turning or twisting action of the force .When you apply a force to an object with the purpose of turning that object, you are applying a torque.
- The SI unit of torque is the newton-meter (N·m).
- **Caution:** The newton-meter is also the unit of work. Torque and work, however, are quite different quantities and must not be confused. Work is often expressed in joules (1 J =1 N·m), but torque never is.

12-3~12-5 Electric Field

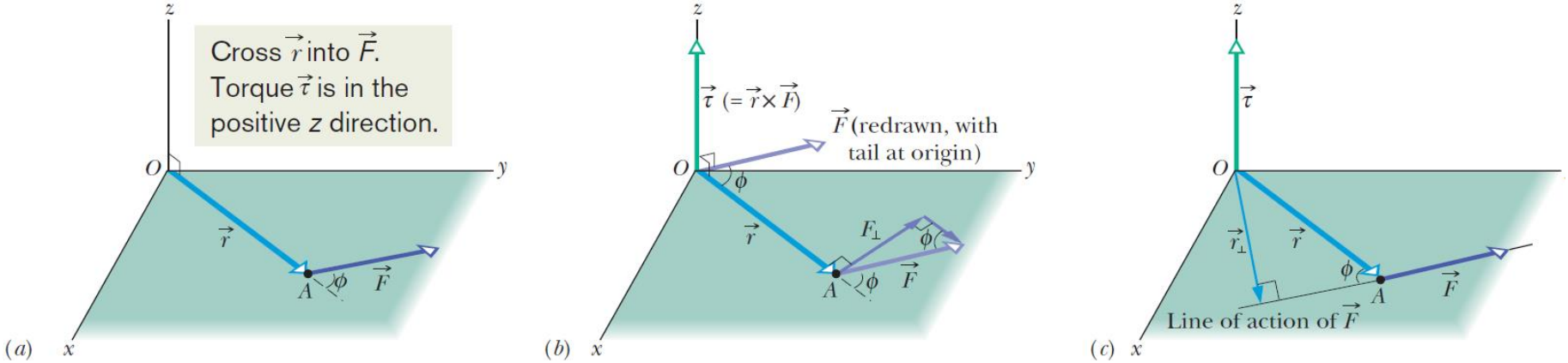
- Torques obey the superposition principle for forces: When several torques act on a body, the *net torque* (or *resultant torque*) is the sum of the individual torques. The symbol for net torque is τ_{net}.
- Despite defining torque τ for a rigid body that can rotate around a fixed axis. We can also expand the definition of torque to apply it to an individual particle that moves along any path relative to a fixed *point* (rather than a fixed axis).
- For that, the path need no longer be a circle, and we must write the torque as a vector $\vec{\tau}$ that may have any direction.
- Figure a (on the next page) shows such a particle at point A in an xy plane. A single force $\vec{F}$ in that plane acts on the particle, and the particle's position relative to the origin O is given by position vector $\vec{r}$. The torque acting on the particle relative to the fixed point O is a vector quantity defined as

$$\vec{\tau} = \vec{r} \times \vec{F} \quad \text{(torque defined).}$$

- The magnitude of $\vec{\tau}$ is

$$\tau = rF \sin \phi$$

12-3~12-5 Electric Field



- From Fig. b, the magnitude of $\vec{\tau}$ can be rewritten as

$$\tau = rF_{\perp},$$

where $F_{\perp}\ (= F \sin\phi)$ is the component of $\vec{F}$ perpendicular to $\vec{r}$.

- From Fig. c, it can also be rewritten as

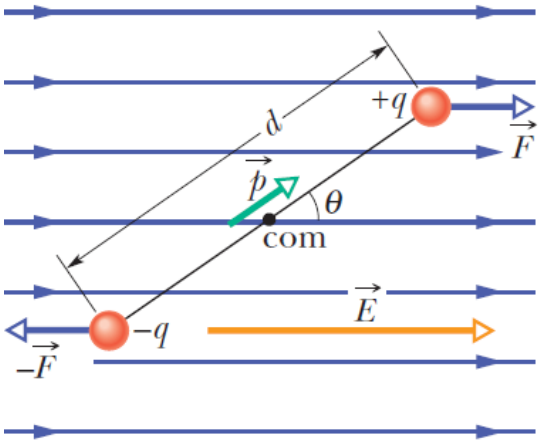
$$\tau = r_{\perp}F,$$

where $r_{\perp}\ (= r \sin\phi)$ is the moment arm of $\vec{F}$ (the perpendicular distance between O and the line of action of $\vec{F}$).

12-3~12-5 Electric Field

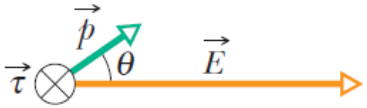
A Dipole in an Electric Field

- We consider a dipole in a uniform external electric field $\vec{E}$, as shown in Fig. 12-21a (also the right). The dipole consists of two opposite charge, each of magnitude q, separated by a distance d. The dipole moment $\vec{p}$ makes an angle θ with field $\vec{E}$.
- Because the electric field is uniform, the electrostatic forces act in opposite directions, as shown, and with the same magnitude $F = qE$. Thus, because *the field is uniform*, the net force on the dipole from the field is zero and the center of mass of the dipole does not move.
- However, the forces on the charged ends do produce a net torque $\vec{\tau}$ on the dipole about its center of mass.



(a)

The dipole is being torqued into alignment.



(b)

12-3~12-5 Electric Field

- The center of mass lies on the line connecting the charged ends, at some distance x from one end and thus a distance $d - x$ from the other end. From Eq. $\tau = rF \sin \phi$, we can write the magnitude of the net torque $\vec{\tau}$ as
- $$\tau = Fx \sin \theta + F(d - x) \sin \theta = Fd \sin \theta. \quad (12\text{-}40)$$
- We can also write the magnitude of $\vec{\tau}$ in terms of the magnitudes of the electric field $\vec{E}$ and the dipole moment $p = qd$. To do so, we substitute qE for F and p/q for d in Eq. 12-40, finding that the magnitude of $\vec{\tau}$ is
- $$\tau = pE \sin \theta. \quad (12\text{-}41)$$
- We can generalize this equation to vector form as
- $$\vec{\tau} = \vec{p} \times \vec{E} \quad \text{(torque on a dipole).} \quad (12\text{-}42)$$
- These vectors are shown in Fig. 12-21b.
- The torque acting on a dipole tends to clockwise rotate $\vec{p}$ into the direction of field $\vec{E}$. We can represent a torque that gives rise to a clockwise rotation by including a minus sign with the magnitude of the torque
- $$\tau = -pE \sin \theta. \quad (12\text{-}43)$$

12-3~12-5 Electric Field

Potential Energy of an Electric Dipole

- Potential energy can be associated with the orientation of an electric dipole in an electric field. The dipole has its least potential energy when it is in its equilibrium orientation, which is when its moment $\vec{p}$ is lined up with the field $\vec{E}$. It has greater potential energy in all other orientations.
- Thus the dipole is like a pendulum, which has *its (the pendulum's)* least gravitational potential energy in *its* equilibrium orientation—at its lowest point. To rotate the dipole or the pendulum to any other orientation requires work by some external agent.
- To calculate the potential energy, here we choose it to be zero when the angle θ in Fig. 12-21 is 90°. We then can find the potential energy U of the dipole at any other value of θ with Eq. $\Delta U = -W$ by calculating the work W done by the field on the dipole when the dipole is rotated to that value of θ from 90°.With the aid of Eq. $W = \int \tau \, d\theta$ and Eq. 12-43,
- $$U = -W = -\int_{90°}^{\theta} \tau \, d\theta = \int_{90°}^{\theta} pE \sin\theta \, d\theta. \qquad (12\text{-}44)$$

12-3~12-5 Electric Field

- Evaluating the integral leads to
- $$U = -pE\cos\theta. \quad (12\text{-}45)$$
- It can be generalized to vector form as
- $$U = -\vec{p}\cdot\vec{E} \quad \text{(potential energy of a dipole).} \quad (12\text{-}46)$$
- Equations 12-45 and 12-46 show us that the potential energy of the dipole is least ($U = -pE$) when $\theta = 0$ ($\vec{p}$ and $\vec{E}$ are in the same direction); the potential energy is greatest ($U = pE$) when $\theta = 180°$ ($\vec{p}$ and $\vec{E}$ are in opposite directions).
- When a dipole rotates from an initial orientation θ_i to another orientation θ_f, the work W done on the dipole by the electric field is
- $$W = -\Delta U = -(U_f - U_i), \quad (12\text{-}47)$$

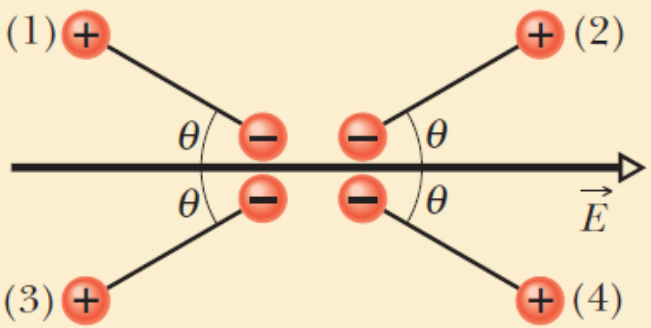
  where U_f and U_i can be calculated with Eq. 12-46.
- If the change in orientation is caused by an applied torque (due to an external agent), then the work W_a done on the dipole by the applied torque is the negative of the work done on the dipole by the field; that is,
- $$W_a = -W = (U_f - U_i). \quad (12\text{-}48)$$

12-3~12-5 Electric Field

- *CHECKPOINT 3* on page 381 of the textbook.

Checkpoint

The figure shows four orientations of an electric dipole in an external electric field. Rank the orientations according to (a) the magnitude of the torque on the dipole and (b) the potential energy of the dipole, greatest first.



12-3~12-5 Electric Field

Review & Summary

Electric Field To explain the electrostatic force between two charges, we assume that each charge sets up an electric field in the space around it. The force acting on each charge is then due to the electric field set up at its location by the other charge.

Definition of Electric Field The *electric field* $\vec{E}$ at any point is defined in terms of the electrostatic force $\vec{F}$ that would be exerted on a positive test charge q_0 placed there:

$$\vec{E} = \frac{\vec{F}}{q_0}.$$

Electric Field Lines *Electric field lines* provide a means for visualizing the direction and magnitude of electric fields. The electric field vector at any point is tangent to a field line through that point. The density of field lines in any region is proportional to the magnitude of the electric field in that region. Field lines originate on positive charges and terminate on negative charges.

Field Due to a Point Charge The magnitude of the electric field $\vec{E}$ set up by a point charge q at a distance r from the charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}.$$

The direction of $\vec{E}$ is away from the point charge if the charge is positive and toward it if the charge is negative.

Field Due to an Electric Dipole An *electric dipole* consists of two particles with charges of equal magnitude q but opposite sign, separated by a small distance d. Their **electric dipole moment** $\vec{p}$ has magnitude qd and points from the negative charge to the positive charge. The magnitude of the electric field set up by the dipole at a distant point on the dipole axis (which runs through both charges) is

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3},$$

where z is the distance between the point and the center of the dipole.

Field Due to a Continuous Charge Distribution The electric field due to a *continuous charge distribution* is found by treating charge elements as point charges and then summing, via integration, the electric field vectors produced by all the charge elements to find the net vector.

12-3~12-5 Electric Field

Field Due to a Charged Disk The electric field magnitude at a point on the central axis through a uniformly charged disk is given by

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right),$$

where z is the distance along the axis from the center of the disk, R is the radius of the disk, and σ is the surface charge density.

Force on a Point Charge in an Electric Field When a point charge q is placed in an external electric field $\vec{E}$, the electrostatic force $\vec{F}$ that acts on the point charge is

$$\vec{F} = q\vec{E}.$$

Force $\vec{F}$ has the same direction as $\vec{E}$ if q is positive and the opposite direction if q is negative.

Dipole in an Electric Field When an electric dipole of dipole moment $\vec{p}$ is placed in an electric field $\vec{E}$, the field exerts a torque $\vec{\tau}$ on the dipole:

$$\vec{\tau} = \vec{p} \times \vec{E}.$$

The dipole has a potential energy U associated with its orientation in the field:

$$U = -\vec{p} \cdot \vec{E}.$$

This potential energy is defined to be zero when $\vec{p}$ is perpendicular to $\vec{E}$; it is least ($U = -pE$) when $\vec{p}$ is aligned with $\vec{E}$ and greatest ($U = pE$) when $\vec{p}$ is directed opposite $\vec{E}$.

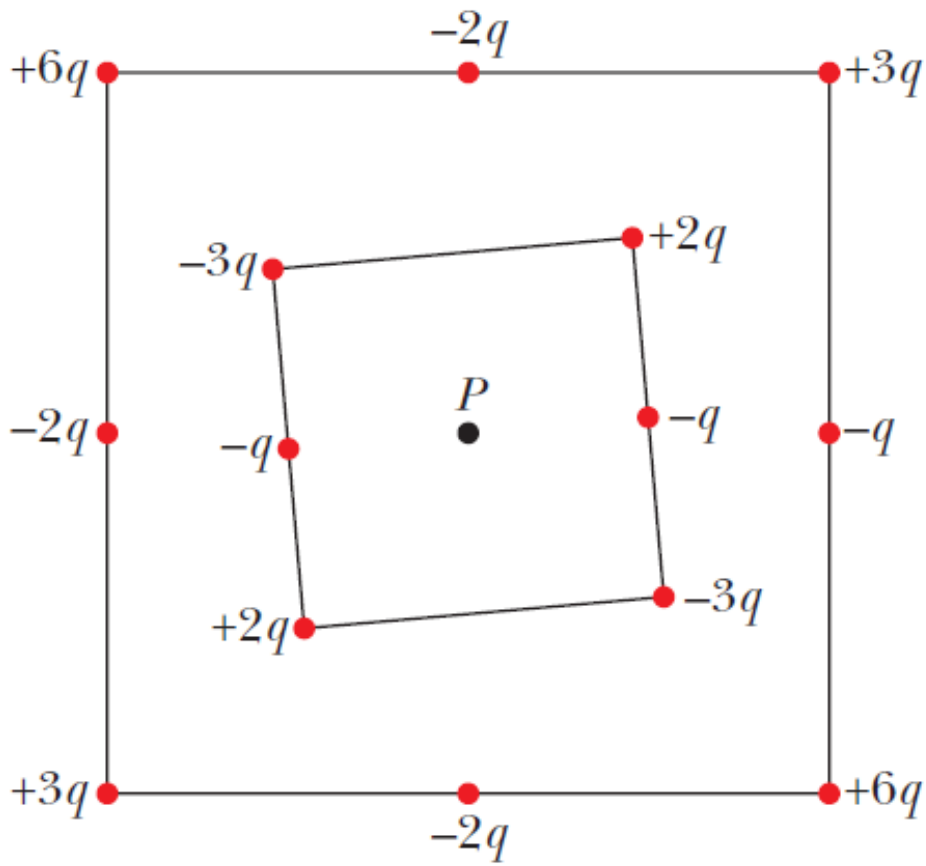
Questions to Answer

- Questions 4, 5, and 6 on page 396 of the textbook.

12-1~12-2 Electric Charge

Question-4 (page 396)

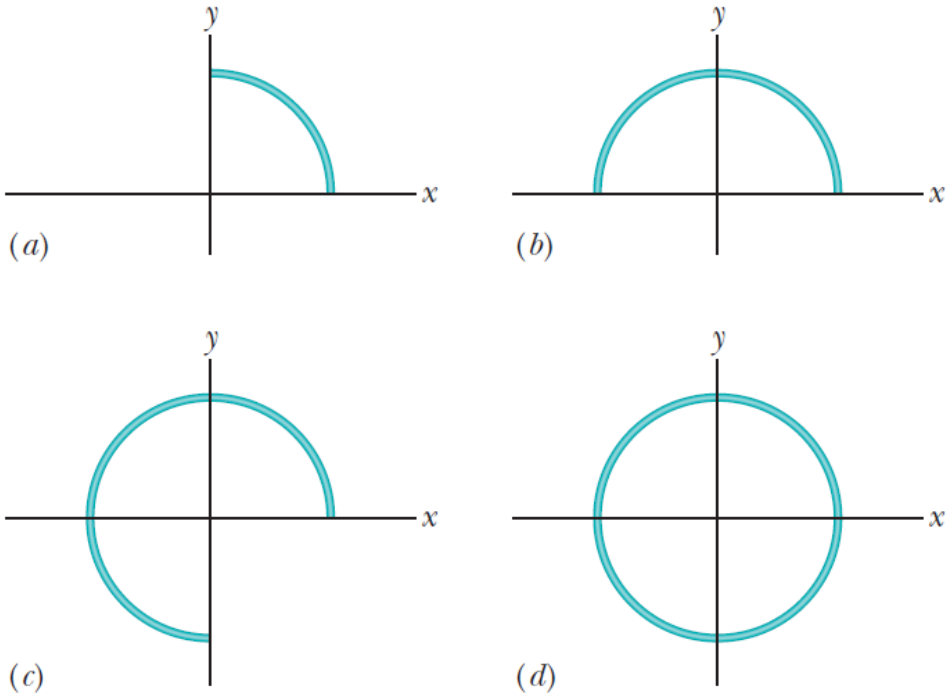
- Figure below shows two square arrays of charged particles. The squares, which are centered on point P, are misaligned. The particles are separated by either d or $d/2$ along the perimeters of the squares. What are the magnitude and direction of the net electric field at P?



12-1~12-2 Electric Charge

Question-5 (page 396)

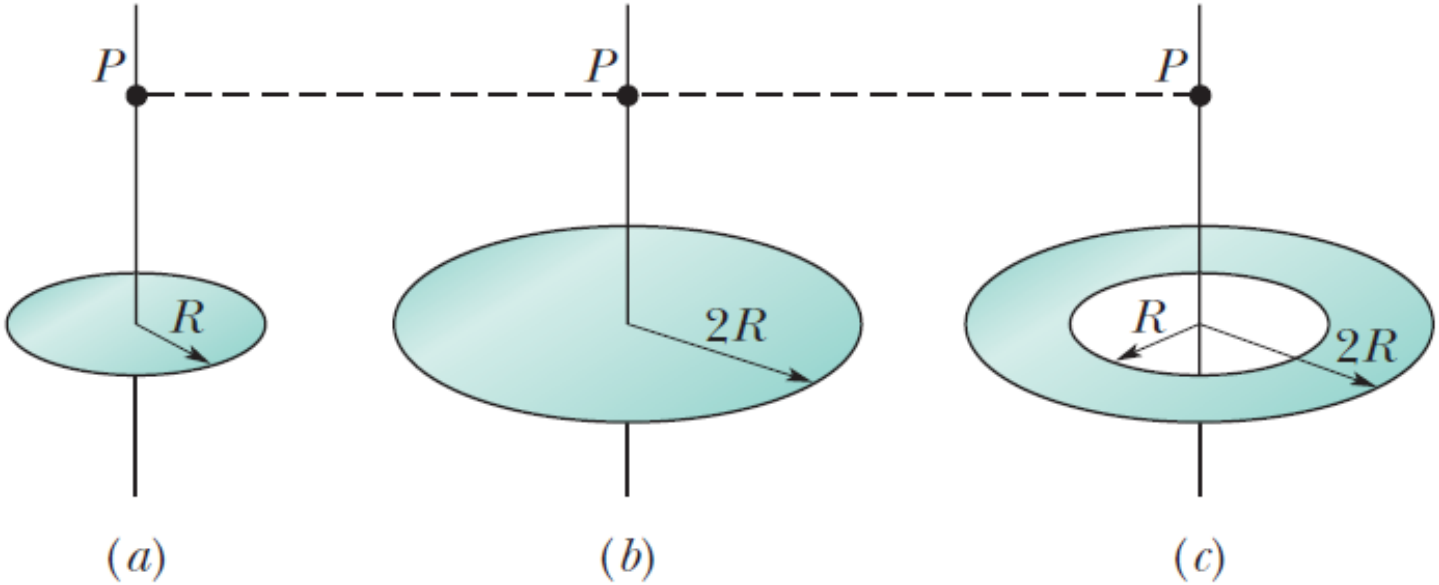
- In Figure below (a), a circular plastic rod with uniform charge $+Q$ produces an electric field of magnitude E at the center of curvature (at the origin). In Figs. (b), (c), and (d), more circular rods, each with identical uniform charges $+Q$, are added until the circle is complete. A fifth arrangement (which would be labeled (e)) is like that in (d) except the rod in the fourth quadrant has charge $-Q$. Rank the five arrangements according to the magnitude of the electric field at the center of curvature, greatest first.



12-1~12-2 Electric Charge

Question-6 (page 396)

- Figure below shows two disks and a flat ring, each with the same uniform charge Q. Rank the objects according to the magnitude of the electric field they create at points P (which are at the same vertical heights), greatest first.



12-6~12-9 Electric Potential

- **What is Physics?**

- One goal of physics is to identify basic forces in our world, such as the electric force.
- A related goal is to determine whether a force is *conservative—that is, whether a potential energy can be associated with it*.
- The motivation for associating a potential energy with a force is that we can then apply the principle of the conservation of mechanical energy to closed systems involving the force. This extremely powerful principle allows us to calculate the results of experiments for which force calculations alone would be very difficult.
- Experimentally, physicists and engineers discovered that the ***electric force is conservative and thus has an associated electric potential energy*** .

12-6~12-9 Electric Potential

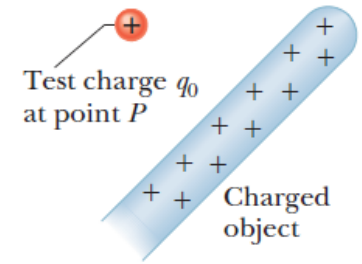
- We are going to define the electric potential (or potential for short) in terms of electric potential energy, so *our first job is to figure out how to measure that potential energy.*
- Back in Chapter 4, we measured gravitational potential energy U of an object by (1) assigning $U = 0$ for a reference configuration (such as the object at table level) and (2) then calculating the work W the gravitational force does if the object is moved up or down from that level. We then defined the potential energy as being

$$U = -W \quad \text{(potential energy)}.$$

- We can follow the same procedure with our new conservative force, the electric force.

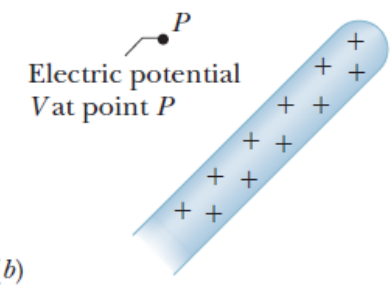
12-6~12-9 Electric Potential

- In Fig. a (right), we want to find the potential energy U associated with a positive test charge q_0 located at point P in the electric field of a charged rod.
- First, we need a reference configuration for which $U = 0$. reasonable choice is for the test charge to be **infinitely far** from the rod, because then there is no interaction with the rod.
- Next, we bring the test charge in from infinity to point P to form the configuration of Fig. a. The work and thus the potential energy can be positive or negative depending on the sign of the rod's charge.
- We define the ***electric potential*** V at P in terms of the work done by the electric force W_∞ and the resulting potential energy:
- $$V = \frac{-W_\infty}{q_0} = \frac{U}{q_0} \quad \text{(electric potential).} \quad (12\text{-}52)$$



(a)

The rod sets up an electric potential, which determines the potential energy.



(b)

12-6~12-9 Electric Potential

- That is, ***the electric potential is the amount of electric potential energy per unit charge when a positive test charge is brought in from infinity.***
- The rod sets up this potential V at P *regardless of whether the test charge (or anything else) happens to be there* (Fig. b on the last page).
- V is a scalar quantity (because there is no direction associated with potential energy or charge) and can be positive or negative (because potential energy and charge have signs).
- In fact, every charged object sets up electric potential V at points throughout its electric field. If we happen to place a particle with, say, charge q at a point where we know the pre-existing V, we can immediately find the potential energy of the configuration:

$$(\text{electric potential energy}) = (\text{particle's charge})\left(\frac{\text{electric potential energy}}{\text{unit charge}}\right),$$

or

$$U = qV,$$

where q can be positive or negative.

12-6~12-9 Electric Potential

- **Two Cautions.** (1) The potential V and potential energy *are related but they are different and not interchangeable*. (2) Electric potential is a ***scalar***, not a vector.
- **Language.** A potential energy is a ***property of a system (or configuration) of objects***, but we can get away with assigning it to a single object. If a charged particle is placed in an electric field and has no noticeable effect on the field, we usually *assign the electric potential energy to the particle alone*. This is similar to the case of gravitational potential energy or elastic potential energy .
- **Units.** The SI unit for potential is the joule per coulomb. This combination occurs so often that a special unit, the volt (abbreviated V), is used to represent it. Thus,
- $$1 \text{ volt} = 1 \text{ joule per coulomb.} \tag{12-53}$$
- With two unit conversions, we can now switch the unit for electric field from newtons per coulomb to a more conventional unit:
- $$1 \text{ N/C} = \left(1\frac{\text{N}}{\text{C}}\right)\left(\frac{1 \text{ V}}{1 \text{ J/C}}\right)\left(\frac{1 \text{ J}}{1 \text{ N}\cdot\text{m}}\right) = 1 \text{ V/m.} \tag{12-54}$$
- From now on, we shall express values of the electric field in ***volts per meter*** rather than in newtons per coulomb.

12-6~12-9 Electric Potential

Motion Through an Electric Field

- **Change in Electric Potential.** If we move from an initial point i to a second point f in the electric field of a charged particle, the electric potential changes by

$$\Delta V = V_f - V_i.$$

- If the particle has charge q then the potential energy of the system changes by

$$\Delta U = q\,\Delta V = q(V_f - V_i).$$

- The change can be positive or negative, depending on the signs of q and ΔV It can also be zero, if there is no change in potential from i to f .
- Because the electric force is conservative, the change in potential energy ΔU between i and f is the same for all paths between those points, i.e. ***path independent***.
- **Work by the Field.**

$$W = -\Delta U = -q\,\Delta V = -q(V_f - V_i).$$

- Up until now, we have always attributed work to a force but here can also say W is the work done on the particle by the electric field (because it, of course, produces the force).

12-6~12-9 Electric Potential

- **Conservation of Energy.** If a charged particle moves through an electric field with *no force acting on it other than the electric force* due to the field, then the *mechanical energy is conserved*.
- $U_i + K_i = U_f + K_f,$ or $\Delta K = -\Delta U.$
- Then $$\Delta K = -q\,\Delta V = -q(V_f - V_i).$$
- **Work by an Applied Force.** If *some force in addition to the electric force* acts on the particle, we say that the additional force is an *applied force* or *external force* , which is often attributed to an *external agent* . Such an applied force can do work on the particle, but the force may not be conservative and thus, in general, we cannot associate a potential energy with it. We account for that work W_{app} by

$$(\text{initial energy}) + (\text{work by applied force}) = (\text{final energy})$$

or $$U_i + K_i + W_{\text{app}} = U_f + K_f.$$

- It can also be written as

$$\Delta K = -\Delta U + W_{\text{app}} = -q\,\Delta V + W_{\text{app}}.$$

- The work by the applied force can be positive, negative, or zero, and thus the energy of the system can increase, decrease, or remain the same.

12-6~12-9 Electric Potential

- In the special case where the particle is stationary before and after the move, the kinetic energy terms are zero and we have
- $$W_{\text{app}} = q\,\Delta V \quad (\text{for } K_i = K_f). \tag{12-58}$$
- In this special case, the work W_{app} involves the motion of the particle through the potential difference ΔV and not a change in the particle's kinetic energy.
- In this special case, the work by the applied force is the negative of the work by the field:
- $$W_{\text{app}} = -W \quad (\text{for } K_i = K_f). \tag{12-56}$$
- **Electron-volts.** In atomic and subatomic physics, energy measures in the SI unit of joules often require awkward powers of ten. A more convenient (but non-SI unit) is the ***electron-volt*** (eV), which is defined to be equal to the work required to move a single elementary charge e (such as that of an electron or proton) through a potential difference ΔV of exactly one volt. Thus

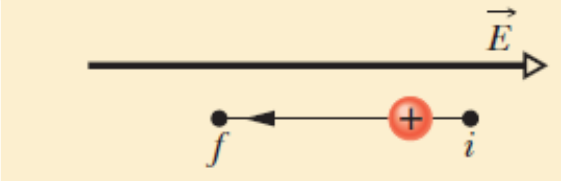
$$1\ \text{eV} = e(1\ \text{V})$$

$$= (1.602 \times 10^{-19}\ \text{C})(1\ \text{J/C}) = 1.602 \times 10^{-19}\ \text{J}.$$

12-6~12-9 Electric Potential

- *CHECKPOINT 4* on page 383 of the textbook

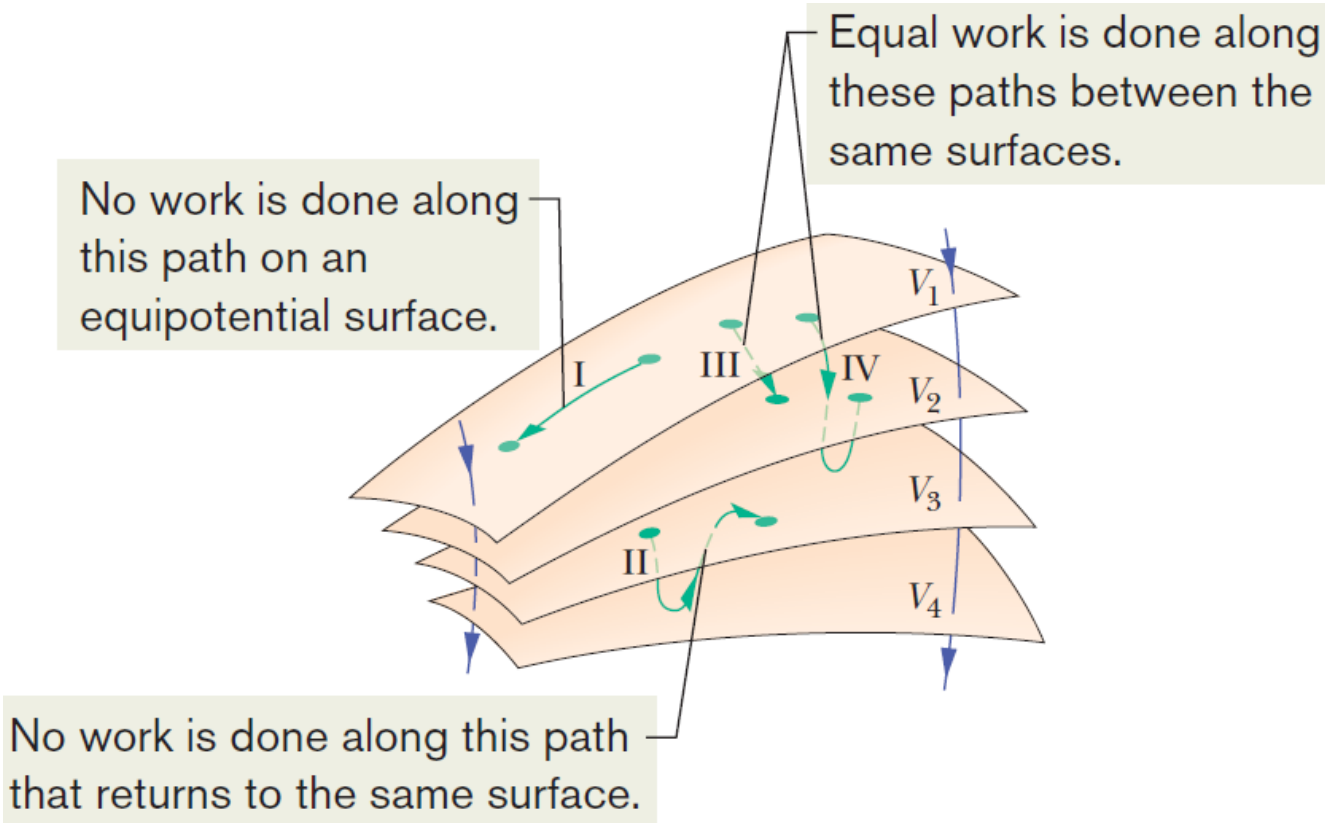
In the figure, we move a proton from point *i* to point *f* in a uniform electric field. Is positive or negative work done by (a) the electric field and (b) our force? (c) Does the electric potential energy increase or decrease? (d) Does the proton move to a point of higher or lower electric potential?



12-6~12-9 Electric Potential

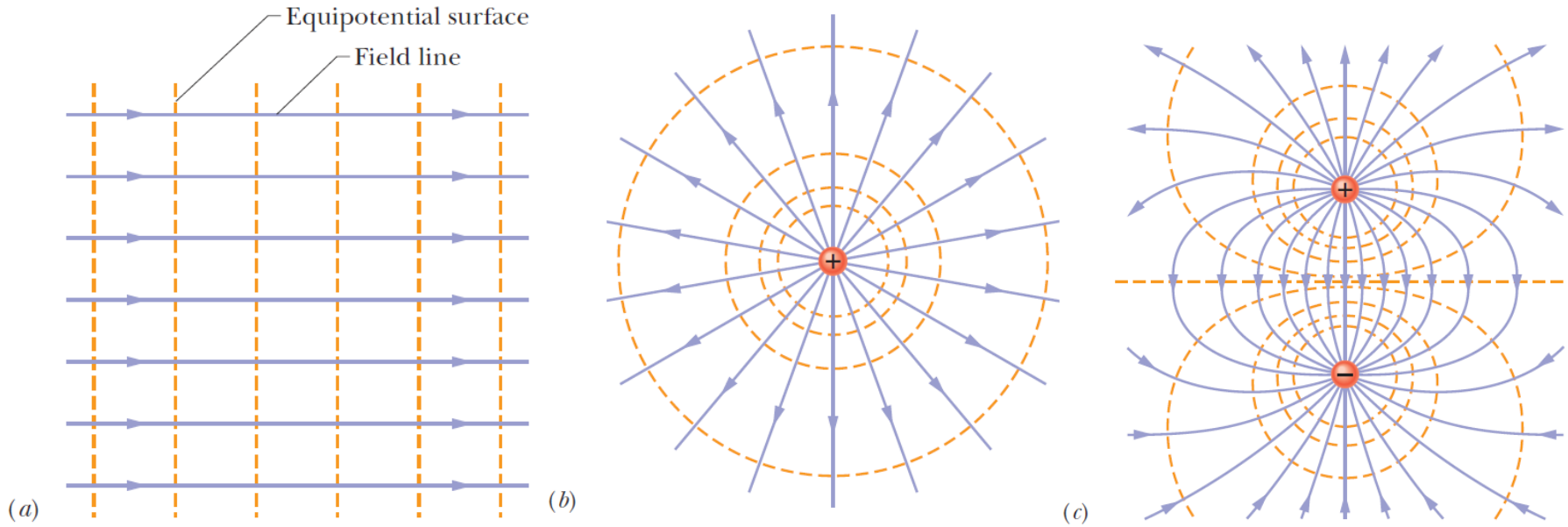
- **Equipotential Surfaces**

- Adjacent points that have the same electric potential form an ***equipotential surface,*** which can be either an imaginary surface or a real, physical surface
- Figure below shows a family of equipotential surfaces associated with the electric field due to some distribution of charges.



12-6~12-9 Electric Potential

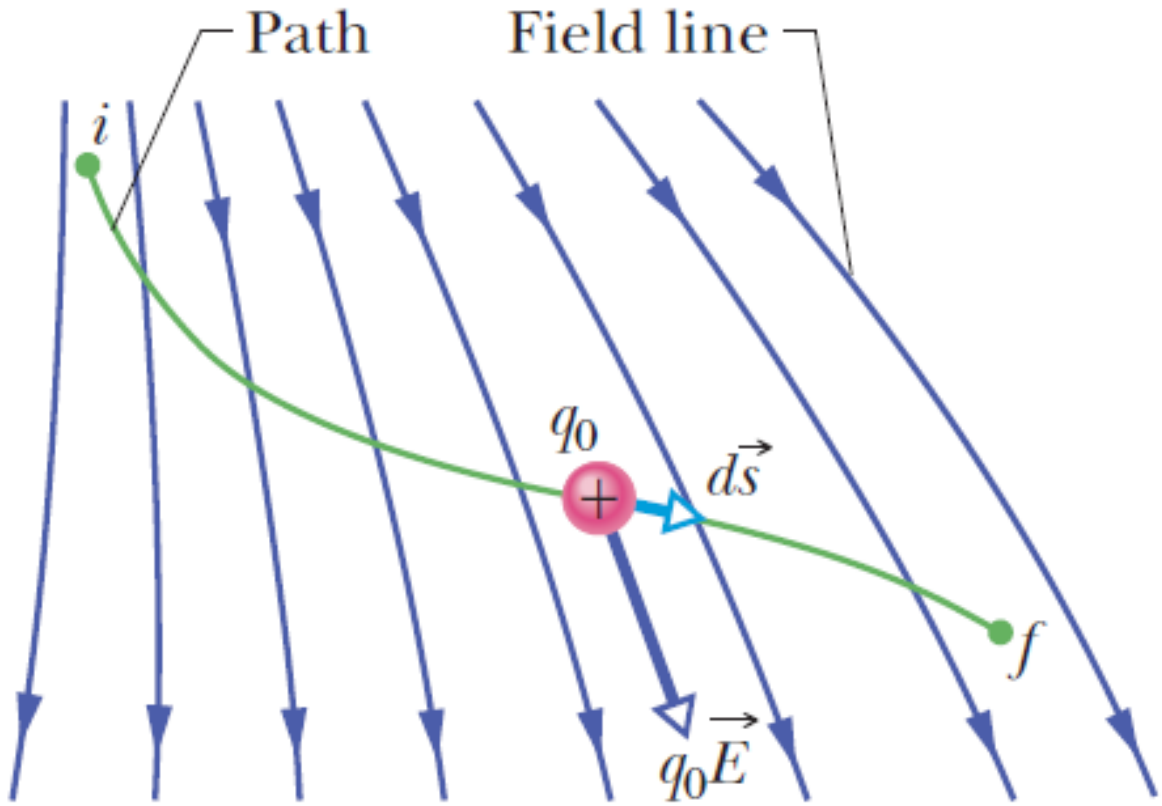
- Figure below shows electric field lines and cross sections of the equipotential surfaces for a ***uniform electric field (a)*** and for the ***field associated with a charged particle (b) and with an electric dipole (c)*** .



12-6~12-9 Electric Potential

- **Calculating the Potential from the Field**

- Consider an arbitrary electric field, represented by the field lines in Fig. 12-24 (also below), and a positive test charge q_0 that moves along the path shown from point i to point f.



12-6~12-9 Electric Potential

- At any point on the path, an electric force $q_0\vec{E}$ acts on the charge as it moves through a differential displacement $d\vec{s}$. Then the differential work dW is
- $$dW = q_0\vec{E}\cdot d\vec{s}. \qquad (12\text{-}60)$$
- To find the total work W done on the particle by the field as the particle moves from point i to point f, we sum—via integration—the differential works done on the charge as it moves through all the displacements $d\vec{s}$ along the path:
- $$W = q_0\int_i^f \vec{E}\cdot d\vec{s}. \qquad (12\text{-}61)$$
- If we substitute the total work W from Eq. 12-61 into the equation $W = -\Delta U = -q\,\Delta V = -q(V_f - V_i)$, we find

$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}. \qquad (12\text{-}62)$$

- Thus, the potential difference between any two points in an electric field is equal to the negative of the line integral (meaning the integral along a particular path). Because the electric force is conservative, all paths yield the same result.

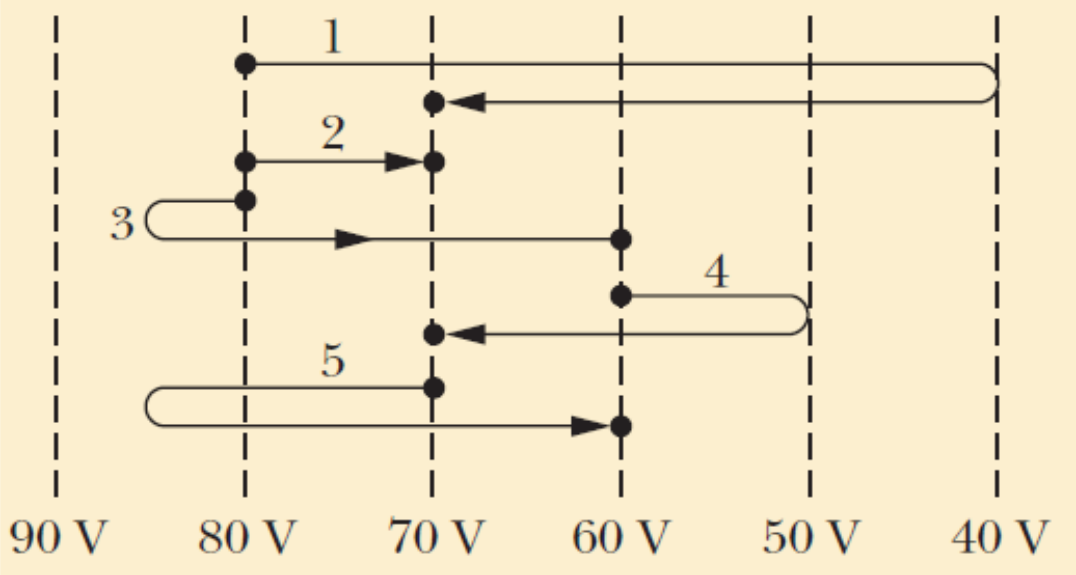
12-6~12-9 Electric Potential

- Equation 12-62 allows us to calculate the difference in potential between any two points in the field. If we set potential $V_i = 0$, then Eq. 12-62 becomes
- $$V = -\int_i^f \vec{E} \cdot d\vec{s}, \qquad (12\text{-}63)$$
- in which we have dropped the subscript f on V_f . Equation 12-63 gives us the potential V at any point f in the electric field ***relative to the zero potential*** at point i . If we let point i be at infinity, then Eq. 12-63 gives us the potential V at any point f relative to the zero potential at infinity.
- ***CHECKPOINT 5*** on page 385 of the textbook.

12-6~12-9 Electric Potential

- *CHECKPOINT 5* on page 385 of the textbook

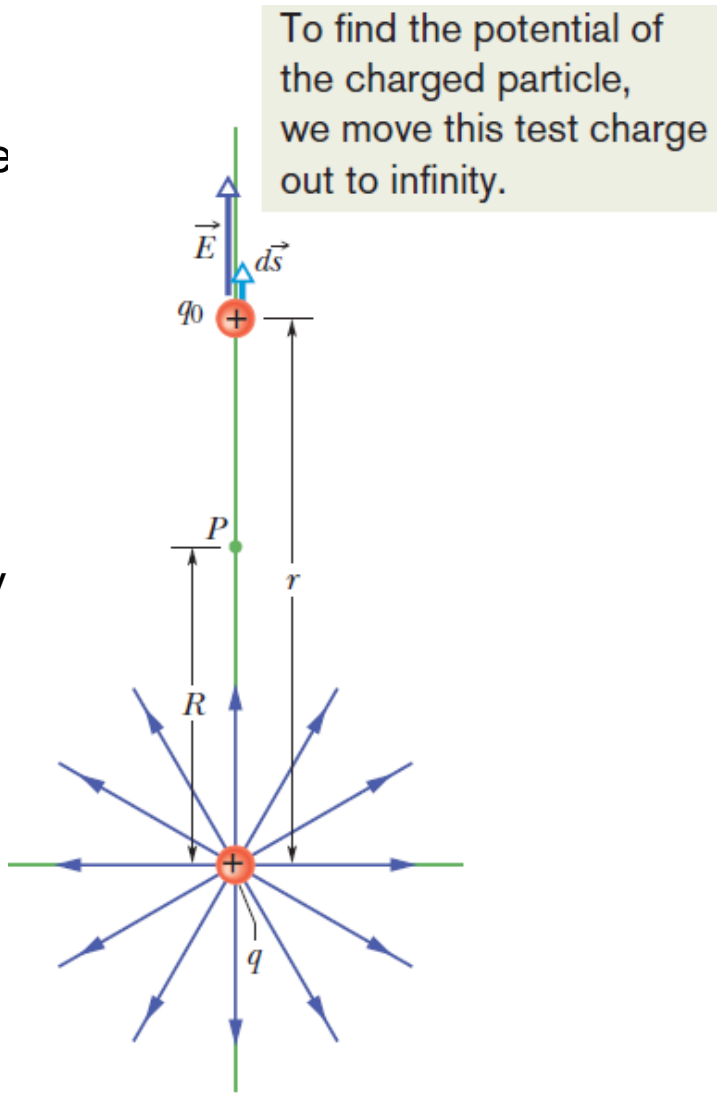
The figure here shows a family of parallel equipotential surfaces (in cross section) and five paths along which we shall move an electron from one surface to another. (a) What is the direction of the electric field associated with the surfaces? (b) For each path, is the work we do positive, negative, or zero? (c) Rank the paths according to the work we do, greatest first.



12-6~12-9 Electric Potential

- **Potential Due to a Charged Particle**

- Consider a point P at distance R from a fixed particle of positive charge q (Fig. 12-26, also the right).
- To use Eq. 12-62, we imagine that we move a positive test charge q_0 from point P to infinity. Because the path we take does not matter, let us choose the simplest one— a line that extends radially from the fixed particle through P to infinity
- To use Eq. 12-62, we must evaluate the dot product
- $$\vec{E} \cdot d\vec{s} = E \cos\theta \, ds. \qquad (12\text{-}66)$$
- Angle $\theta = 0$ and $\cos\theta = 1$. Because the path is radial, let us write ds as dr. Then, substituting the limits R and ∞, we can write Eq. 12-62 as



12-6~12-9 Electric Potential

- $$V_f - V_i = -\int_R^\infty E\,dr. \qquad (12\text{-}67)$$
- Next, we set $V_f = 0$ (at ∞) and $V_i = V$ (at R). Then, for the magnitude of the electric field at the site of the test charge, we substitute from Eq. 12-14:
- $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}. \qquad (12\text{-}68)$$
- Then, $$0 - V = -\frac{q}{4\pi\varepsilon_0}\int_R^\infty \frac{1}{r^2}\,dr = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_R^\infty$$
- $$= -\frac{1}{4\pi\varepsilon_0}\frac{q}{R}. \qquad (12\text{-}69)$$
- Solving for V and switching R to r, we then have
- $$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad (12\text{-}70)$$

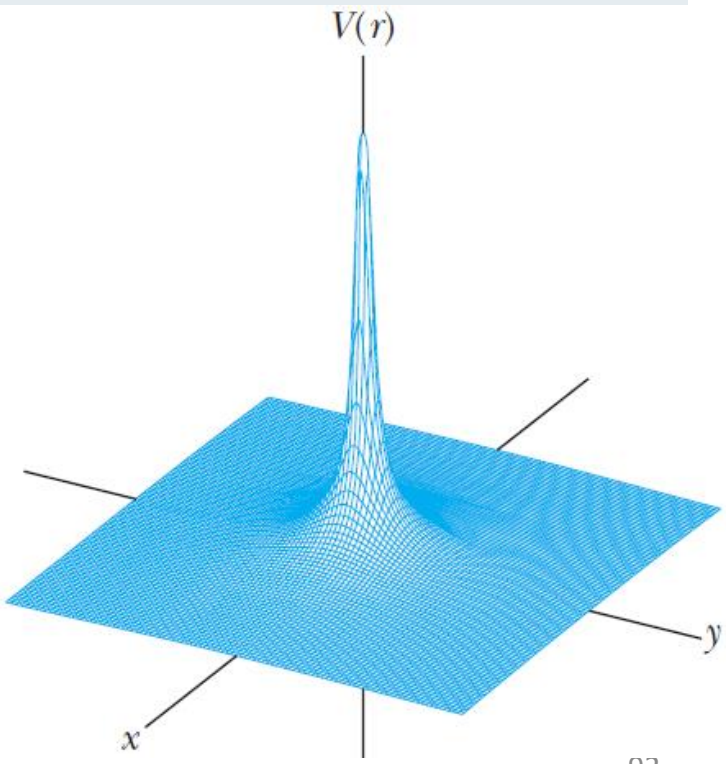
as the electric potential V due to a particle of charge q at any radial distance r from the particle.

12-6~12-9 Electric Potential

- Although we have derived Eq. 12-70 for a positively charged particle, the derivation holds also for a negatively charged particle, in which case, q is a negative quantity. Note that the sign of V is the same as the sign of q :

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential.

- Figure 12-27 (also the right) shows a computer-generated plot of Eq. 12-70 for a positively charged particle; the magnitude of V is plotted vertically.
- Note that the magnitude increases as $r \rightarrow 0$. In fact, according to Eq. 12-70, V is infinite at $r = 0$, although Fig. 12-27 shows a finite, smoothed-off value there.



12-6~12-9 Electric Potential

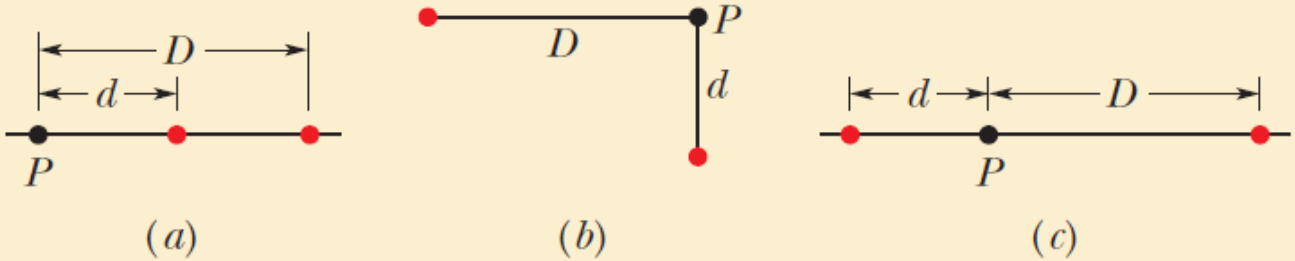
- **Potential Due to a Group of Charged Particles**

- We can find the net electric potential at a point due to a group of charged particles with the help of the ***superposition principle*** . Using Eq. 12-70 with the plus or minus sign of the charge included, we calculate separately the potential resulting from each charge at the given point. Then we sum the potentials. Thus, for n charges, the net potential is
- $$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i} \quad (n \text{ charged particles}). \qquad (12\text{-}71)$$
- The sum in Eq. 12-71 is an ***algebraic sum*** , not a vector sum.
- ***CHECKPOINT 6*** on page 387 of the textbook.
- ***Sample Problem 12-7*** on the pages of 387 and 388 of the textbook.

12-6~12-9 Electric Potential

- *CHECKPOINT 6* on page 387 of the textbook

The figure here shows three arrangements of two protons. Rank the arrangements according to the net electric potential produced at point P by the protons, greatest first.



12-6~12-9 Electric Potential

- *Sample Problem 12-7* on the pages of 387 and 388 of the textbook

Sample Problem 24.04 **Potential is not a vector, orientation is irrelevant**

(a) In Fig. 24-12*a*, 12 electrons (of charge $-e$) are equally spaced and fixed around a circle of radius R. Relative to $V = 0$ at infinity, what are the electric potential and electric field at the center C of the circle due to these electrons?

KEY IDEAS

(1) The electric potential V at C is the algebraic sum of the electric potentials contributed by all the electrons. Because electric potential is a scalar, the orientations of the electrons do not matter. (2) The electric field at C is a vector quantity and thus the orientation of the electrons *is* important.

Potential is a scalar and orientation is irrelevant.

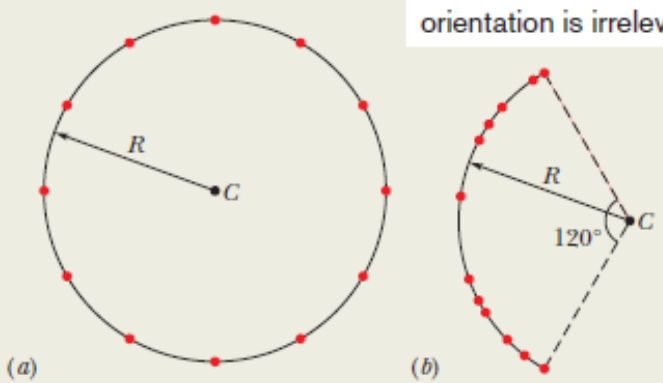


Figure 24-12 (*a*) Twelve electrons uniformly spaced around a circle. (*b*) The electrons nonuniformly spaced along an arc of the original circle.

Calculations: Because the electrons all have the same negative charge $-e$ and are all the same distance R from C, Eq. 24-27 gives us

$$V = -12\frac{1}{4\pi\varepsilon_0}\frac{e}{R}. \qquad \text{(Answer)} \qquad (24\text{-}28)$$

Because of the symmetry of the arrangement in Fig. 24-12*a*, the electric field vector at C due to any given electron is canceled by the field vector due to the electron that is diametrically opposite it. Thus, at C,

$$\vec{E} = 0. \qquad \text{(Answer)}$$

(b) The electrons are moved along the circle until they are nonuniformly spaced over a 120° arc (Fig. 24-12*b*). At C, find the electric potential and describe the electric field.

Reasoning: The potential is still given by Eq. 24-28, because the distance between C and each electron is unchanged and orientation is irrelevant. The electric field is no longer zero, however, because the arrangement is no longer symmetric. A net field is now directed toward the charge distribution.

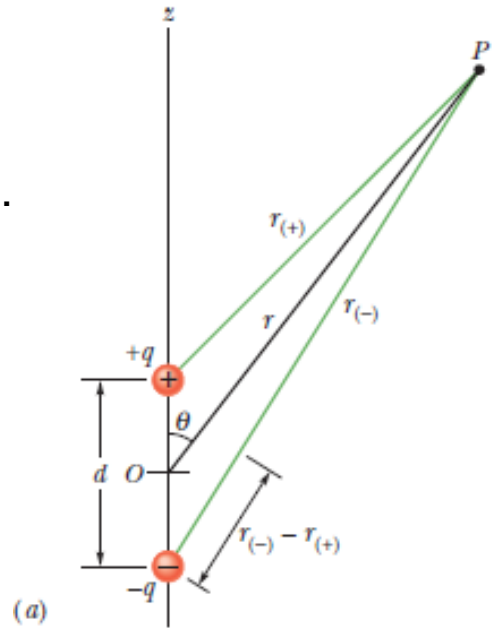
12-6~12-9 Electric Potential

- **Potential Due to an Electric Dipole**

- Now let us apply Eq. 12-71 to an electric dipole to find the potential at an arbitrary point P in Fig. 12-29a (also the right).

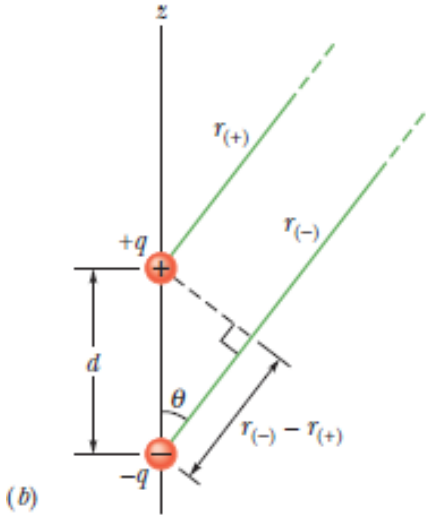
$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right)$$

- $$= \frac{q}{4\pi\varepsilon_0}\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}. \qquad (12\text{-}73)$$



- Naturally occurring dipoles—such as those possessed by many molecules—are quite small; so we are usually interested only in points that are relatively far from the dipole, such that $r \gg d$.

- In that case, we can approximate the two lines to P as being parallel and their length difference as being the leg of a right triangle with hypotenuse d (Fig. 12-29b). Also, that difference is so small that the product of the lengths is approximately r^2 .Thus,



12-6~12-9 Electric Potential

$$r_{(-)} - r_{(+)} \approx d\cos\theta \quad \text{and} \quad r_{(-)}r_{(+)} \approx r^2.$$

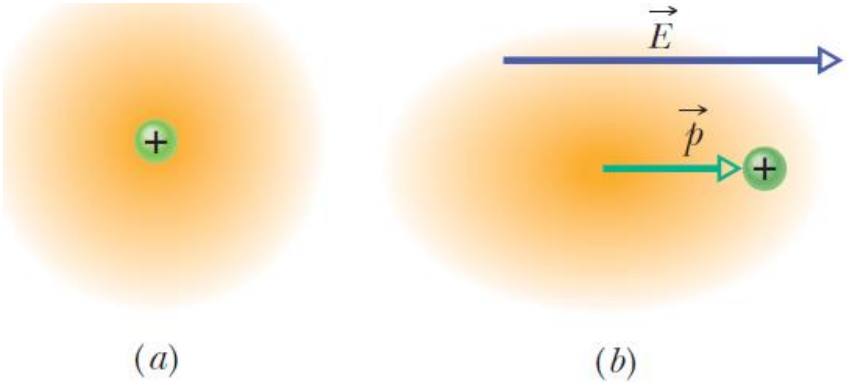
- Substitute these quantities into Eq. 12-73, we can approximate V to be
- $$V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2},$$
- It can also be written as
- $$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \quad \text{(electric dipole)}, \qquad (12\text{-}74)$$

in which $p\ (= qd)$ is the magnitude of the electric dipole moment $\vec{p}$.

12-6~12-9 Electric Potential

- **Induced Dipole Moment**

- Many molecules, such as water, have ***permanent*** electric dipole moments.
- In other molecules (called ***nonpolar molecules***) and in every isolated atom, the centers of the positive and negative charges coincide (Fig. 12-30a, also seen as below) and thus no dipole moment is set up.
- However, if we place an atom or a nonpolar molecule in an external electric field, the field distorts the electron orbits and separates the centers of positive and negative charge (Fig. 12-30b). Because the electrons are negatively charged, they tend to be shifted in a direction opposite the field. This shift sets up a dipole moment that $\vec{p}$ points in the direction of the field. This dipole moment is said to be ***induced*** by the field, and the atom or molecule is then said to be ***polarized*** by the field. When the field is removed, the induced dipole moment and the polarization disappear.



12-6~12-9 Electric Potential

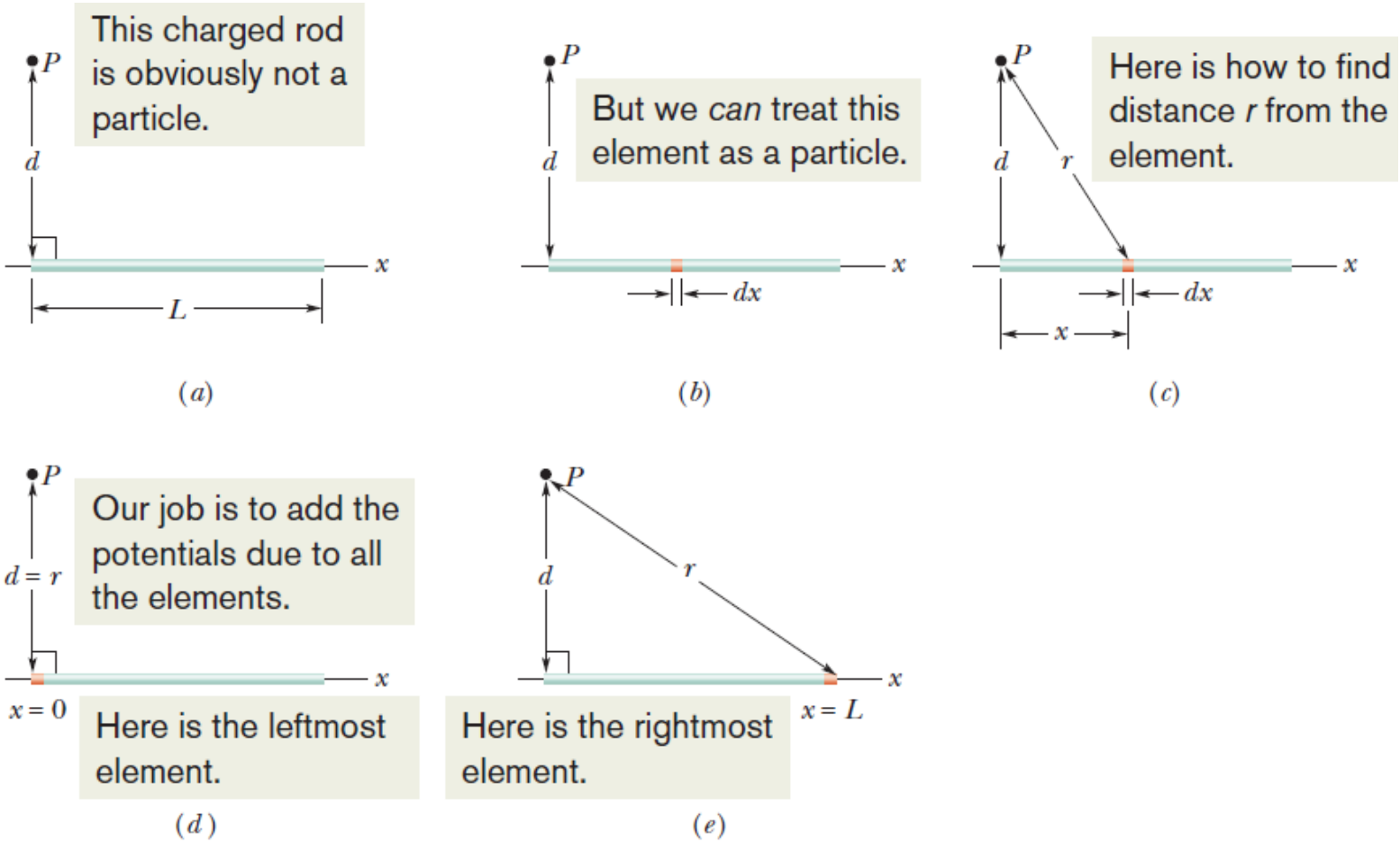
- **Potential Due to a Continuous Charge Distribution**

- When a charge distribution q is continuous (as on a uniformly charged thin rod or disk), we cannot use the summation of Eq. 12-71 to find the potential V at P . Instead, we must choose a differential element of charge dq, determine the potential dV at P due to dq, and then integrate over the entire charge distribution.
- Let us again take the zero of potential to be at infinity. If we treat the element of charge dq as a particle, then we can use Eq. 12-71 to express the potential dV at point P due to dq :
- $$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} \quad \text{(positive or negative } dq\text{).} \qquad (12\text{-}75)$$
- Here r is the distance between P and dq. To find the total potential V at P , we integrate to sum the potentials due to all the charge elements:
- $$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}. \qquad (12\text{-}76)$$
- We now examine two continuous charge distributions, a line and a disk.

12-6~12-9 Electric Potential

- **Line of Charge**

- In Fig. 12-31a (also below), a thin nonconducting rod of length L has a positive charge of uniform linear density λ . Let us determine the electric potential V due to the rod at point P, a perpendicular distance d from the left end of the rod.



12-6~12-9 Electric Potential

- We consider a differential element dx of the rod as shown in Fig. 12-31b.This (or any other) element of the rod has a differential charge of
- $$dq = \lambda\, dx. \qquad (12\text{-}77)$$
- It produces an electric potential dV at P, which is a distance $r = (x^2 + d^2)^{1/2}$ from the element (Fig. c). Treating the element as a point charge, we can use Eq. 12-75 to write the potential dV as
- $$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(x^2 + d^2)^{1/2}}. \qquad (12\text{-}78)$$
- Since the charge on the rod is positive and we have taken $V = 0$ at infinity, we know that dV in Eq. 12-78 must be positive.
- We now find the total potential V produced by the rod at point P by integrating Eq. 12-78 along the length of the rod, from $x = 0$ to $x = L$ (Figs. d and e), using integral 17 in Appendix E. We find

12-6~12-9 Electric Potential

$$V = \int dV = \int_0^L \frac{1}{4\pi\varepsilon_0} \frac{\lambda}{(x^2 + d^2)^{1/2}} dx$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \int_0^L \frac{dx}{(x^2 + d^2)^{1/2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \left[\ln\left(x + (x^2 + d^2)^{1/2} \right) \right]_0^L$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \left[\ln\left(L + (L^2 + d^2)^{1/2} \right) - \ln d \right].$$

- We can simplify this result by using the general relation $\ln A - \ln B = \ln(A/B)$,
- $$V = \frac{\lambda}{4\pi\varepsilon_0} \ln \left[\frac{L + (L^2 + d^2)^{1/2}}{d} \right]. \qquad (12\text{-}79)$$
- Because V is the sum of positive values of dV, it too is positive, consistent with the logarithm being positive for an argument greater than 1.

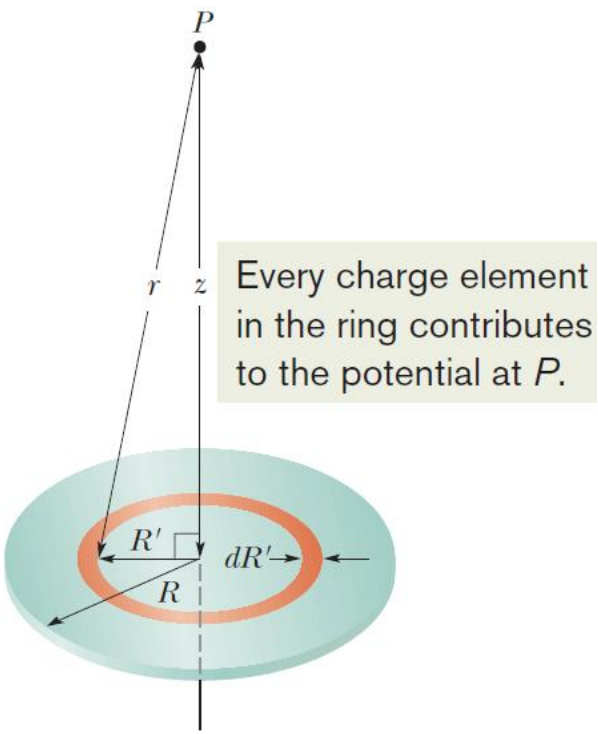
12-6~12-9 Electric Potential

- **Charged Disk**

- In 12-4, we calculated the magnitude of the electric field at points on the central axis of a plastic disk of radius R that has a uniform charge density σ on one surface. Here we derive an expression for $V(z)$, the electric potential at any point on the central axis.
- Because we have a circular distribution of charge on the disk, we could start with a differential element that occupies angle $d\theta$ and radial distance dr. We would then need to set up a two-dimensional integration.
- However, let's do something easier.
- In Fig. 12-32 (also the right), consider a differential element consisting of a flat ring of radius R' and radial width dR'. Its charge has magnitude

$$dq = \sigma(2\pi R')(dR'),$$

in which $(2\pi R')(dR')$ is the upper surface area of the ring.



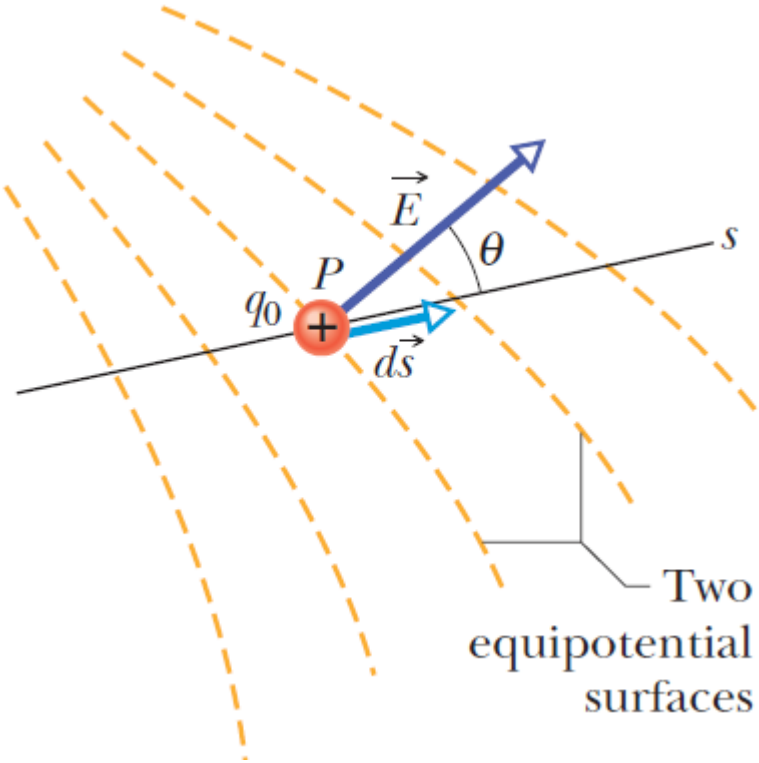
12-6~12-9 Electric Potential

- All parts of this charged element are the same distance r from point P on the disk's axis. With the aid of Fig. 12-32, we can use Eq. 12-75 to write the contribution of this ring to the electric potential at P as
- $$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi R')(dR')}{\sqrt{z^2 + R'^2}}. \quad (12\text{-}80)$$
- We find the net potential at P by adding (via integration) the contributions of all the rings from $R' = 0$ to $R' = R$:
- $$V = \int dV = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{R'\,dR'}{\sqrt{z^2 + R'^2}} = \frac{\sigma}{2\varepsilon_0}(\sqrt{z^2 + R^2} - z). \quad (12\text{-}81)$$
- Note that the variable in the second integral of Eq. 12-81 is R' and not z , which remains constant while the integration over the surface of the disk is carried out.
- Note also that, in evaluating the integral, we have assumed that $z \geq 0$.

12-6~12-9 Electric Potential

- **Calculating the Field from the Potential**

- In previous Module, you saw how to find the potential at a point f if you know the electric field along a path from a reference point to point f. In this module, we propose to go the other way—that is, ***to find the electric field when we know the potential***.
- Figure 12-33 shows cross sections of a family of closely spaced equipotential surfaces, the potential difference between each pair of adjacent surfaces being dV. As the figure suggests, the field at any point P is perpendicular to the equipotential surface through P.
- Suppose that a positive test charge q_0 moves through a displacement $d\vec{s}$ from one equipotential surface to the adjacent surface.



12-6~12-9 Electric Potential

- From Eq. 12-51, the work the electric field does on the test charge during the move is $-q_0\, dV$
- From Eq. 12-60 and Fig. 12-33, we see that the work done by the electric field may also be written as the scalar product $(q_0\vec{E}) \cdot d\vec{s}$, $q_0 E(\cos\theta)\, ds$ ating these two expressions for the work yields
- $$-q_0\, dV = q_0 E(\cos\theta)\, ds, \qquad (12\text{-}82)$$

  or
- $$E\cos\theta = -\frac{dV}{ds}. \qquad (12\text{-}83)$$
- Since $E\cos\theta$ is the component of $\vec{E}$ in the direction of $d\vec{s}$, Eq. 12-83 becomes
- $$E_s = -\frac{\partial V}{\partial s}. \qquad (12\text{-}84)$$
- In words, Eq. 12-84 (which is essentially the reverse operation of Eq. 12-62) states:

> The component of $\vec{E}$ in any direction is the negative of the rate at which the electric potential changes with distance in that direction.

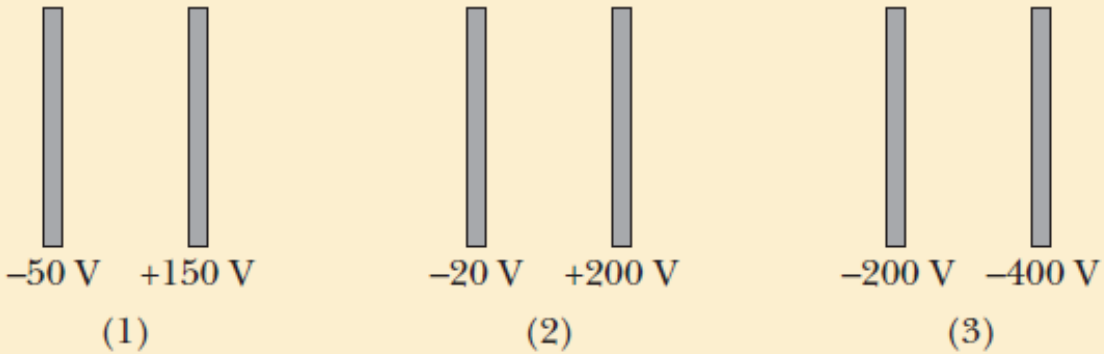
12-6~12-9 Electric Potential

- If we take the s axis to be, in turn, the x, y, and z axes, we find that the x, y, and z components of at any point are
- $$E_x = -\frac{\partial V}{\partial x}; \qquad E_y = -\frac{\partial V}{\partial y}; \qquad E_z = -\frac{\partial V}{\partial z}. \qquad (12\text{-}85)$$
- Thus, if we know V for all points in the region around a charge distribution—that is, if we know the function $V(x, y, z)$ —we can find the components of $\vec{E}$, and thus $\vec{E}$ itself, at any point by ***taking partial derivatives*** .
- For the simple situation in which the electric field is uniform, Eq. 12-84 becomes
- $$E = -\frac{\Delta V}{\Delta s}, \qquad (12\text{-}86)$$
- where s is perpendicular to the equipotential surfaces. The component of the electric field is zero in any direction parallel to the equipotential surfaces because there is no change in potential along the surfaces.
- **CHECKPOINT 8** on page 392 of the textbook.

12-6~12-9 Electric Potential

- *CHECKPOINT 8* on page 392 of the textbook

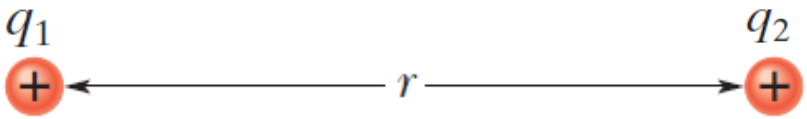
The figure shows three pairs of parallel plates with the same separation, and the electric potential of each plate. The electric field between the plates is uniform and perpendicular to the plates. (a) Rank the pairs according to the magnitude of the electric field between the plates, greatest first. (b) For which pair is the electric field pointing rightward? (c) If an electron is released midway between the third pair of plates, does it remain there, move rightward at constant speed, move leftward at constant speed, accelerate rightward, or accelerate leftward?



12-6~12-9 Electric Potential

- Electric Potential Energy of a System of Charged Particles

- The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from infinite distance.
- Our starting point is to examine *the work we must do (as an external agent) to bring together two charged particles that are initially infinitely far apart and that end up near each other and stationary*.
- If the two particles have the same sign of charge, we must fight against their mutual repulsion. Our work is then positive and results in a positive potential energy for the final two-particle system.
- If, instead, the two particles have opposite signs of charge, our job is easy because of the mutual attraction of the particles. Our work is then negative and results in a negative potential energy for the system.
- Let's follow this procedure to build the two-particle system in Fig. 12-34 (below), where particle 1 (with positive charge q_1) and particle 2 (with positive charge q_2) have separation r.



12-6~12-9 Electric Potential

- We start with particle 2 fixed in place and particle 1 infinitely far away, with an initial potential energy U_i for the two-particle system.
- Next we bring particle 1 to its final position, and then the system's potential energy is U_f.
- Our work changes the system's potential energy by $\Delta U = U_f - U_i$.
- With Eq. $\Delta U = q(V_f - V_i)$, we can relate ΔU to the change in potential through which we move particle 1:
- $$U_f - U_i = q_1(V_f - V_i).$$
- The initial potential energy is $U_i = 0$ because the particles are in the reference configuration. The two potentials in this equation are due to particle 2 and are given by Eq. 12-70:
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r}.$$
- This tells us that when particle 1 is initially at distance $r = \infty$, the potential at its location is $V_i = 0$.When we move it to the final position at distance r, the potential at its location is
$$V_f = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r}.$$

12-6~12-9 Electric Potential

- Substituting these results into Eq. $U_f - U_i = q_1(V_f - V_i)$ and dropping the subscript f, we find that the final configuration has a potential energy of
- $$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \quad \text{(two-particle system).} \qquad (12\text{-}87)$$
- Equation 12-87 includes the signs of the two charges. If the two charges have the same sign, U is positive. If they have opposite signs, U is negative.
- If we next bring in a third particle, with charge q_3, we repeat our calculation, starting with particle 3 at an infinite distance and then bringing it to a final position at distance r_{31} from particle 1 and distance r_{32} from particle 2. At the final position, the potential V_f at the location of particle 3 is the algebraic sum of the potential V_1 due to particle 1 and the potential V_2 of particle 2.When we work out the algebra, we find that

The total potential energy of a system of particles is the sum of the potential energies for every pair of particles in the system.

- This result applies to a system for any given number of particles.

12-6~12-9 Electric Potential

- Now that we have an expression for the potential energy of a system of particles, we can apply the principle of the conservation of energy to the system as expressed in Equation below,

$$(\text{initial energy}) + (\text{work by applied force}) = (\text{final energy})$$

or

$$U_i + K_i + W_{\text{app}} = U_f + K_f\,.$$

- For example, if the system consists of many particles, we might consider the kinetic energy (and the associated escape speed) required of one of the particles to escape from the rest of the particles.
- **Sample Problem 12-9** on page 393 of the textbook.
- **Sample Problem 12-10** on page 394 of the textbook.

12-6~12-9 Electric Potential

- *Sample Problem 12-9* on page 393 of the textbook

Sample Problem 24.06 **Potential energy of a system of three charged particles**

Figure 24-19 shows three charged particles held in fixed positions by forces that are not shown. What is the electric potential energy U of this system of charges? Assume that $d = 12$ cm and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

in which $q = 150$ nC.

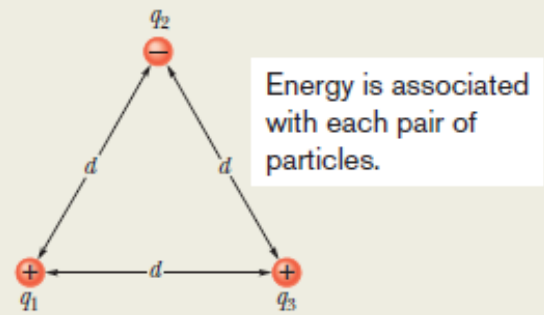


Figure 24-19 Three charges are fixed at the vertices of an equilateral triangle. What is the electric potential energy of the system?

KEY IDEA

The potential energy U of the system is equal to the work we must do to assemble the system, bringing in each charge from an infinite distance.

Calculations: Let's mentally build the system of Fig. 24-19, starting with one of the charges, say q_1, in place and the others at infinity. Then we bring another one, say q_2, in from infinity and put it in place. From Eq. 24-46 with d substituted for r, the potential energy U_{12} associated with the pair of charges q_1 and q_2 is

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d}.$$

We then bring the last charge q_3 in from infinity and put it in place. The work that we must do in this last step is equal to the sum of the work we must do to bring q_3 near q_1 and the work we must do to bring it near q_2. From Eq. 24-46, with d substituted for r, that sum is

$$W_{13} + W_{23} = U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{d} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{d}.$$

The total potential energy U of the three-charge system is the sum of the potential energies associated with the three pairs of charges. This sum (which is actually independent of the order in which the charges are brought together) is

12-6~12-9 Electric Potential

- *Sample Problem 12-9* on page 393 of the textbook

$$
\begin{aligned}
U &= U_{12} + U_{13} + U_{23} \\
&= \frac{1}{4\pi\varepsilon_0}\left(\frac{(+q)(-4q)}{d} + \frac{(+q)(+2q)}{d} + \frac{(-4q)(+2q)}{d}\right) \\
&= -\frac{10q^2}{4\pi\varepsilon_0 d} \\
&= -\frac{(8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2})(10)(150 \times 10^{-9}\ \mathrm{C})^2}{0.12\ \mathrm{m}} \\
&= -1.7 \times 10^{-2}\ \mathrm{J} = -17\ \mathrm{mJ}. \qquad \text{(Answer)}
\end{aligned}
$$

The negative potential energy means that negative work would have to be done to assemble this structure, starting with the three charges infinitely separated and at rest. Put another way, an external agent would have to do 17 mJ of positive work to disassemble the structure completely, ending with the three charges infinitely far apart.

The lesson here is this: If you are given an assembly of charged particles, you can find the potential energy of the assembly by finding the potential of every possible pair of the particles and then summing the results.

12-6~12-9 Electric Potential

- ***Sample Problem 12-10*** on page 394 of the textbook

Sample Problem 24.07 **Conservation of mechanical energy with electric potential energy**

An alpha particle (two protons, two neutrons) moves into a stationary gold atom (79 protons, 118 neutrons), passing through the electron region that surrounds the gold nucleus like a shell and headed directly toward the nucleus (Fig. 24-20). The alpha particle slows until it momentarily stops when its center is at radial distance $r = 9.23$ fm from the nuclear center. Then it moves back along its incoming path. (Because the gold nucleus is much more massive than the alpha particle, we can assume the gold nucleus does not move.) What was the kinetic energy K_i of the alpha particle when it was initially far away (hence external to the gold atom)? Assume that the only force acting between the alpha particle and the gold nucleus is the (electrostatic) Coulomb force and treat each as a single charged particle.

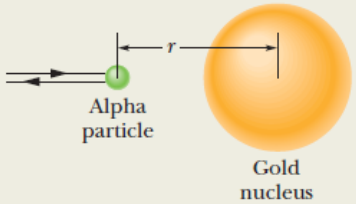


Figure 24-20 An alpha particle, traveling head-on toward the center of a gold nucleus, comes to a momentary stop (at which time all its kinetic energy has been transferred to electric potential energy) and then reverses its path.

KEY IDEA

During the entire process, the mechanical energy of the *alpha particle + gold atom* system is conserved.

Reasoning: When the alpha particle is outside the atom, the system's initial electric potential energy U_i is zero because the atom has an equal number of electrons and protons, which produce a *net* electric field of zero. However, once the alpha particle passes through the electron region surrounding the nucleus on its way to the nucleus, the electric field due to the electrons goes to zero. The reason is that the electrons act like a closed spherical shell of uniform negative charge and, as discussed in Module 23-6, such a shell produces zero electric field in the space it encloses. The alpha particle still experiences the electric field of the protons in the nucleus, which produces a repulsive force on the protons within the alpha particle.

As the incoming alpha particle is slowed by this repulsive force, its kinetic energy is transferred to electric potential energy of the system. The transfer is complete when the alpha particle momentarily stops and the kinetic energy is $K_f = 0$.

Calculations: The principle of conservation of mechanical energy tells us that

$$K_i + U_i = K_f + U_f. \qquad (24\text{-}47)$$

We know two values: $U_i = 0$ and $K_f = 0$. We also know that the potential energy U_f at the stopping point is given by the right side of Eq. 24-46, with $q_1 = 2e$, $q_2 = 79e$ (in which e is the elementary charge, 1.60×10^{-19} C), and $r = 9.23$ fm. Thus, we can rewrite Eq. 24-47 as

$$K_i = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(79e)}{9.23\text{ fm}}$$

$$= \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(158)(1.60 \times 10^{-19}\ \text{C})^2}{9.23 \times 10^{-15}\ \text{m}}$$

$$= 3.94 \times 10^{-12}\ \text{J} = 24.6\ \text{MeV}. \qquad \text{(Answer)}$$

12-6~12-9 Electric Potential

- **Potential of a Charged Isolated Conductor**

An excess charge placed on an isolated conductor will distribute itself on the surface of that conductor so that all points of the conductor—whether on the surface or inside—come to the same potential. This is true even if the conductor has an internal cavity and even if that cavity contains a net charge.

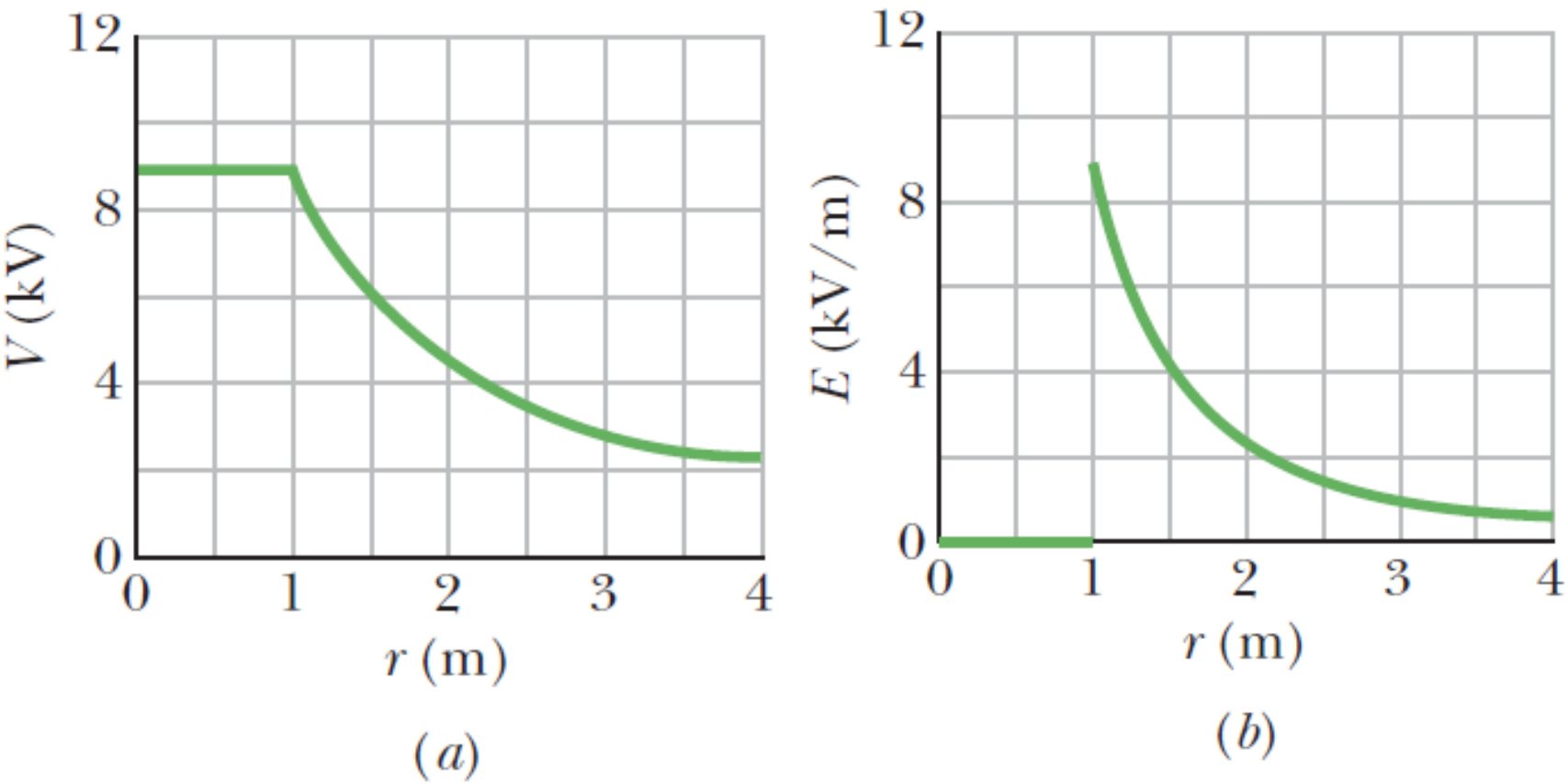
- Our proof follows directly from Eq.

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}.$$

- Since $\vec{E} = 0$ for all points within a conductor, it follows directly that $V_f = V_i$ for all possible pairs of points i and f in the conductor.

12-6~12-9 Electric Potential

- Figure a is a plot of potential against radial distance r from the center for an isolated spherical conducting shell of $1.0\ \mathrm{m}$ radius, having a charge of $1.0\ \mu\mathrm{C}$.
- Figure b shows the variation of electric field with radial distance for the same shell.

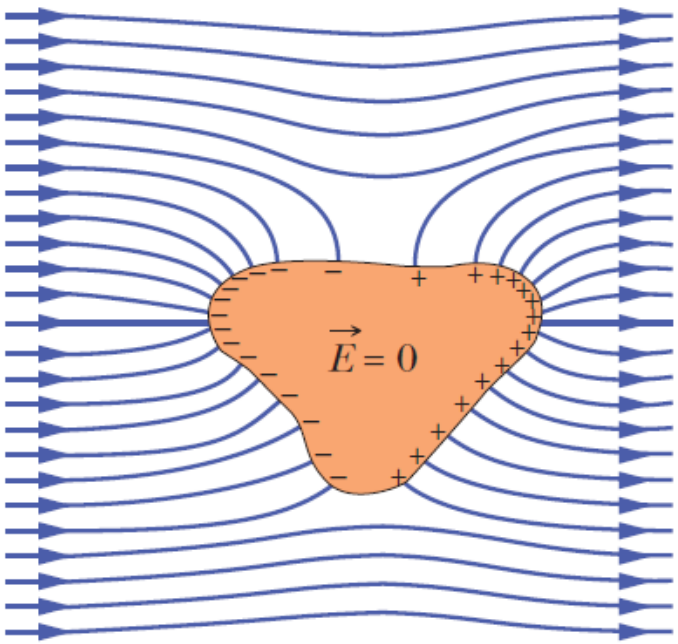


12-6~12-9 Electric Potential

- **Spark Discharge from a Charged Conductor:**
- On nonspherical conductors, a surface charge does not distribute itself uniformly over the surface of the conductor. At sharp points or sharp edges, the surface charge density—and thus the external electric field, which is proportional to it—may reach very high values.
- The air around such sharp points or edges may become ionized, producing the corona discharge that golfers and mountaineers see on the tips of bushes, golf clubs, and rock hammers when thunderstorms threaten. Such corona discharges, like hair that stands on end, are often the precursors of lightning strikes.
- In such circumstances, it is wise to enclose yourself in a cavity inside a conducting shell, where the electric field is guaranteed to be zero. A car (unless it is a convertible or made with a plastic body) is almost ideal (Figure right).

12-6~12-9 Electric Potential

- **Isolated Conductor in an External Electric Field:**
- If an isolated conductor is placed in an external electric field, as in Fig. 12-38 (also the right), all points of the conductor still come to a single potential regardless of whether the conductor has an excess charge.
- Free conduction electrons distribute themselves on the surface in such a way that the electric field they produce at interior points cancels the external electric field that would otherwise be there.
- Furthermore, the electron distribution causes the net electric field at all points on the surface to be perpendicular to the surface.
- If the conductor in Fig. 12-38 could be somehow removed, leaving the surface charges frozen in place, the internal and external electric field would remain absolutely unchanged.



Review & Summary

Review & Summary

Electric Potential The electric potential V at a point P in the electric field of a charged object is

$$V = \frac{-W_\infty}{q_0} = \frac{U}{q_0},$$

where W_∞ is the work that would be done by the electric force on a positive test charge were it brought from an infinite distance to P, and U is the potential energy that would then be stored in the test charge–object system.

Electric Potential Energy If a particle with charge q is placed at a point where the electric potential of a charged object is V, the electric potential energy U of the particle–object system is

$$U = qV.$$

If the particle moves through a potential difference ΔV, the change in the electric potential energy is

$$\Delta U = q\,\Delta V = q(V_f - V_i).$$

Mechanical Energy If a particle moves through a change ΔV in electric potential without an applied force acting on it, applying the conservation of mechanical energy gives the change in kinetic energy as

$$\Delta K = -q\,\Delta V.$$

If, instead, an applied force acts on the particle, doing work W_{app}, the change in kinetic energy is

$$\Delta K = -q\,\Delta V + W_{\text{app}}.$$

In the special case when $\Delta K = 0$, the work of an applied force involves only the motion of the particle through a potential difference:

$$W_{\text{app}} = q\,\Delta V \quad (\text{for } K_i = K_f).$$

Equipotential Surfaces The points on an **equipotential surface** all have the same electric potential. The work done on a test charge in moving it from one such surface to another is independent of the locations of the initial and final points on these surfaces and of the path that joins the points. The electric field $\vec{E}$ is always directed perpendicularly to corresponding equipotential surfaces.

Finding V from $\vec{E}$ The electric potential difference between two points i and f is

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s},$$

where the integral is taken over any path connecting the points. If the integration is difficult along any particular path, we can choose a different path along which the integration might be easier. If we choose $V_i = 0$, we have, for the potential at a particular point,

$$V = -\int_i^f \vec{E} \cdot d\vec{s}.$$

In the special case of a uniform field of magnitude E, the potential change between two adjacent (parallel) equipotential lines separated by distance Δx is

$$\Delta V = -E\,\Delta x.$$

Review & Summary

Potential Due to a Charged Particle The electric potential due to a single charged particle at a distance r from that particle is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r},$$

where V has the same sign as q. The potential due to a collection of charged particles is

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}.$$

Potential Due to an Electric Dipole At a distance r from an electric dipole with dipole moment magnitude $p = qd$, the electric potential of the dipole is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

for $r \gg d$; the angle θ is defined in Fig. 24-13.

Potential Due to a Continuous Charge Distribution For a continuous distribution of charge, Eq. 24-27 becomes

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r},$$

in which the integral is taken over the entire distribution.

Calculating $\vec{E}$ from V The component of $\vec{E}$ in any direction is the negative of the rate at which the potential changes with distance in that direction:

$$E_s = -\frac{\partial V}{\partial s}. \qquad (24\text{-}40)$$

The x, y, and z components of $\vec{E}$ may be found from

$$E_x = -\frac{\partial V}{\partial x}; \qquad E_y = -\frac{\partial V}{\partial y}; \qquad E_z = -\frac{\partial V}{\partial z}.$$

When $\vec{E}$ is uniform, Eq. 24-40 reduces to

$$E = -\frac{\Delta V}{\Delta s},$$

where s is perpendicular to the equipotential surfaces.

Electric Potential Energy of a System of Charged Particles The electric potential energy of a system of charged particles is equal to the work needed to assemble the system with the particles initially at rest and infinitely distant from each other. For two particles at separation r,

$$U = W = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}.$$

Potential of a Charged Conductor An excess charge placed on a conductor will, in the equilibrium state, be located entirely on the outer surface of the conductor. The charge will distribute itself so that the following occur: (1) The entire conductor, including interior points, is at a uniform potential. (2) At every internal point, the electric field due to the charge cancels the external electric field that otherwise would have been there. (3) The net electric field at every point on the surface is perpendicular to the surface.

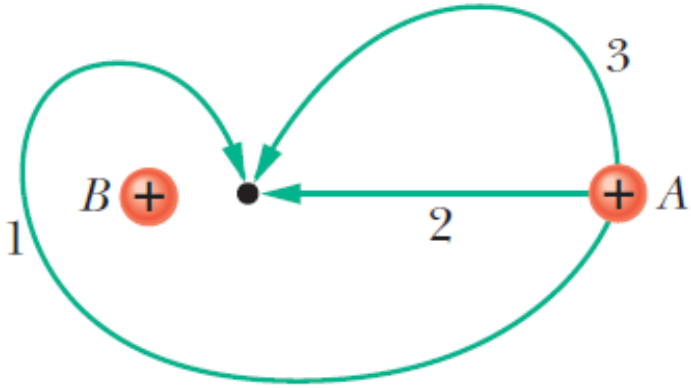
Questions to Answer

- **Questions 7, 8, 9, and 10** on pages 397-398.

12-1~12-2 Electric Charge

Question-7 (page 397)

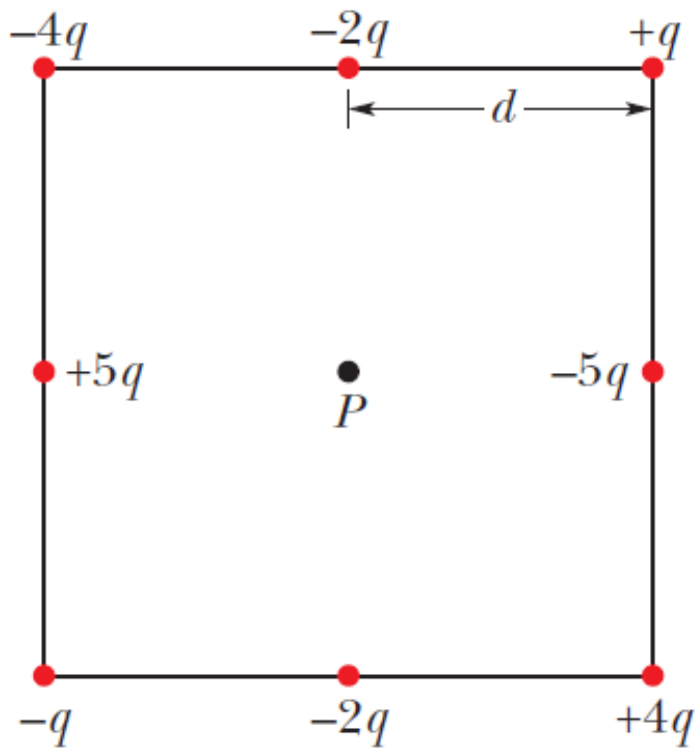
- Figure below shows three paths along which we can move the positively charged sphere A closer to positively charged sphere B, which is held fixed in place. (a) Would sphere A be moved to a higher or lower electric potential? Is the work done (b) by our force and (c) by the electric field due to B positive, negative, or zero? (d) Rank the paths according to the work our force does, greatest first.



12-1~12-2 Electric Charge

Question-8 (page 397)

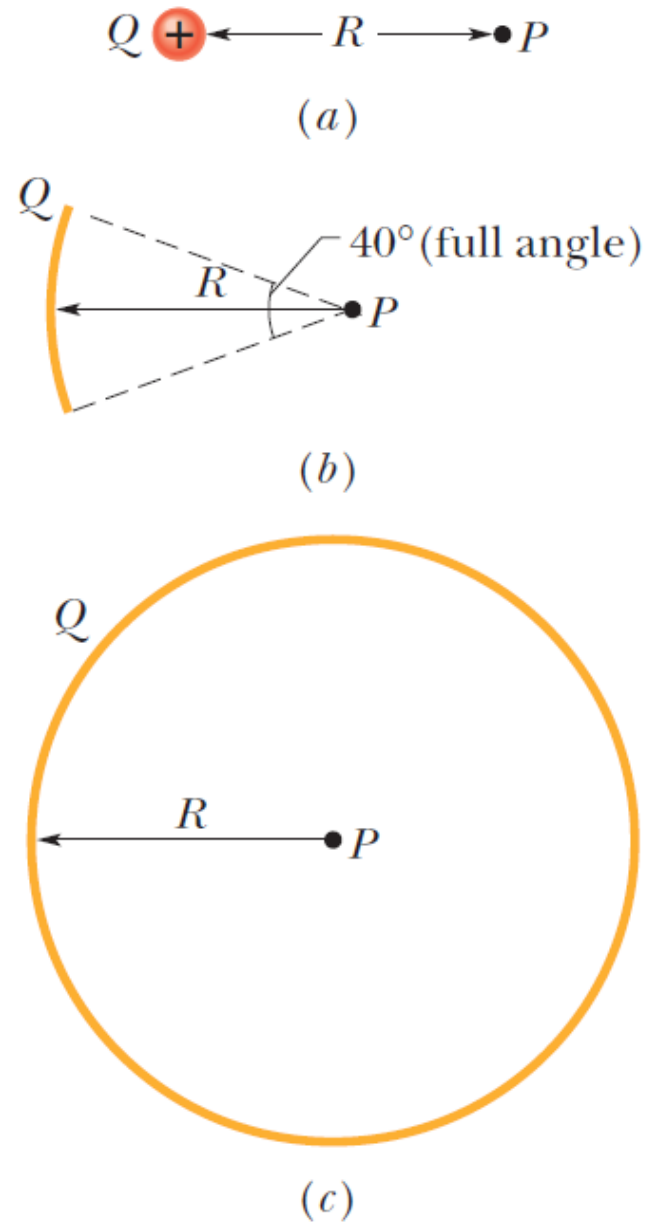
- Figure below shows eight particles that form a square, with distance d between adjacent particles. What is the net electric potential at point P at the center of the square if we take the electric potential to be zero at infinity?



12-1~12-2 Electric Charge

Question-9 (page 397)

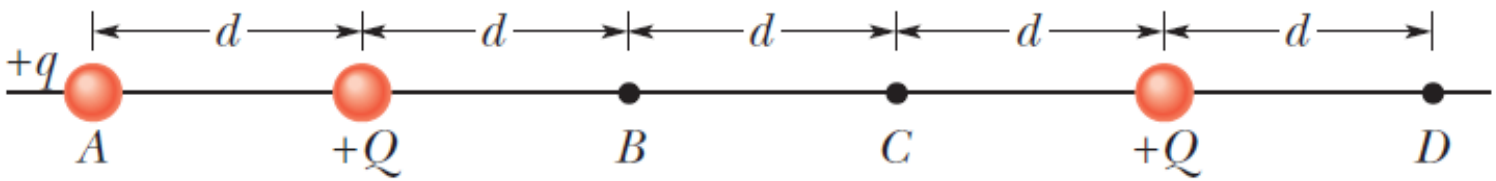
- (a) In Figure right (a), what is the potential at point P due to charge Q at distance R from P ? Set $V = 0$ at infinity. (b) In Fig. (b), the same charge Q has been spread uniformly over a circular arc of radius R and central angle $40°$. What is the potential at point P , the center of curvature of the arc? (c) In Fig. (c), the same charge Q has been spread uniformly over a circle of radius R . What is the potential at point P , the center of the circle? (d) Rank the three situations according to the magnitude of the electric field that is set up at P , greatest first.



12-1~12-2 Electric Charge

Question-10 (page 397)

- Figure below shows a system of three charged particles. If you move the particle of charge $+q$ from point A to point D, are the following quantities positive, negative, or zero: (a) the change in the electric potential energy of the three-particle system, (b) the work done by the net electric force on the particle you moved (that is, the net force due to the other two particles), and (c) the work done by your force? (d) What are the answers to (a) through (c) if, instead, the particle is moved from B to C?



Assignments

- **Assignment-1**

- Problem 1 on page 398, Problem 12 on page 399, and Problem 17 on page 400

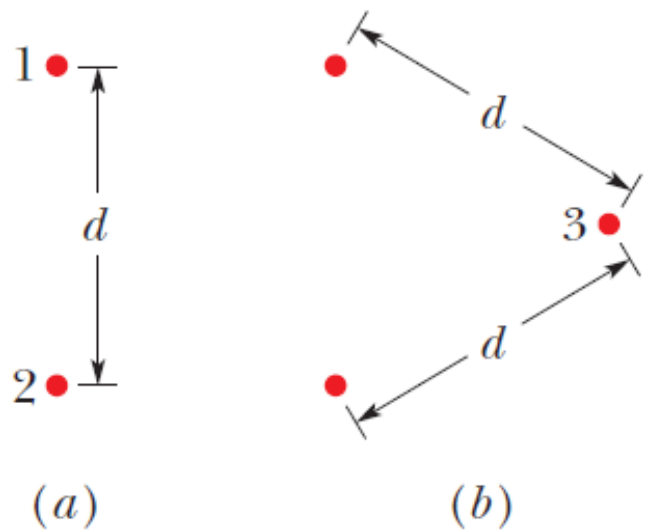
- **Assignment-2**

- Problem 32 on page 402, and Problem 37 on page 403

Assignment-1

Problem-1 (page 398)

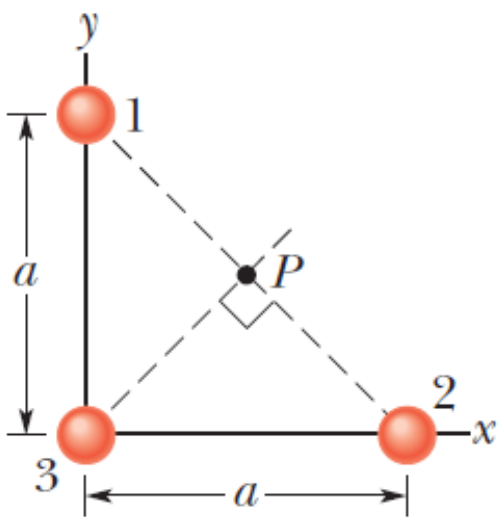
- In Figure (a) below, particles 1 and 2 have charge $20.0\ \mu\mathrm{C}$ each and are held at separation distance $d = 1.50\ \mathrm{m}$. (a) What is the magnitude of the electrostatic force on particle 1 due to particle 2? In Figure (b) below, particle 3 of charge $20.0\ \mu\mathrm{C}$ is positioned so as to complete an equilateral triangle. (b) What is the magnitude of the net electrostatic force on particle 1 due to particles 2 and 3?



Assignment-1

Problem-12 (page 399)

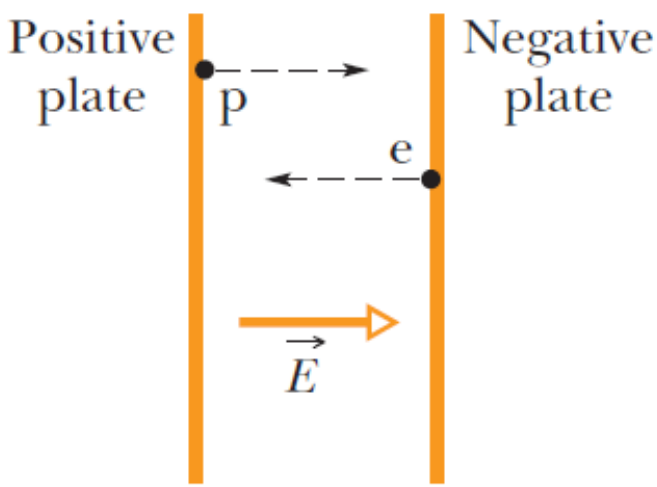
- In Figure below, the three particles are fixed in place and have charges $q_1 = q_2 = +e$ and $q_3 = +2e$. Distance $a = 6.00\ \mu\text{m}$. What are the (a) magnitude and (b) direction of the net electric field at point P due to the particles?



Assignment-1

Problem-17 (page 400)

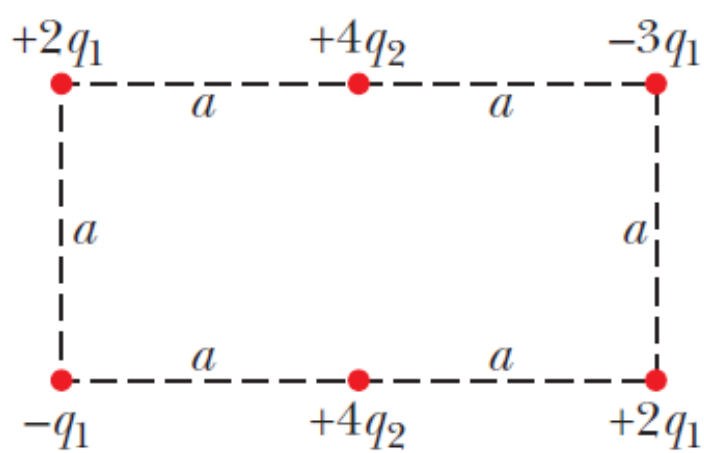
- Two large parallel copper plates are 5.0 cm apart and have a uniform electric field between them as depicted in Figure below. An electron is released from the negative plate at the same time that a proton is released from the positive plate. Neglect the force of the particles on each other and find their distance from the positive plate when they pass each other. (Does it surprise you that you need not know the electric field to solve this problem?)



Assignment-2

Problem-32 (page 402)

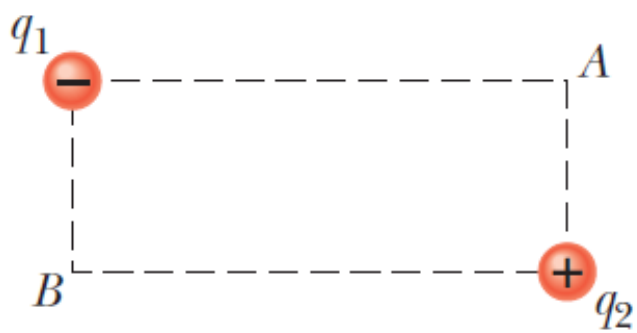
- Figure below shows a rectangular array of charged particles fixed in place, with distance $a = 39.0\ \text{cm}$ and the charges shown as integer multiples of $q_1 = 3.40\ \text{pC}$ and $q_2 = 6.00\ \text{pC}$.With $V = 0$ at infinity, what is the net electric potential at the rectangle's center? (*Hint:* Thoughtful examination of the arrangement can reduce the calculation.)



Assignment-2

Problem-37 (page 403)

- In the rectangle of Figure below, the sides have lengths $5.0\ \text{cm}$ and $15\ \text{cm}$, $q_1 = -5.0\ \mu\text{C}$, $q_2 = +2.0\ \mu\text{C}$. With $V = 0$ at infinity, what is the electric potential at (a) corner A and (b) corner B ? (c) How much work is required to move a charge $q_3 = +3.0\ \mu\text{C}$ from B to A along a diagonal of the rectangle? (d) Does this work increase or decrease the electric potential energy of the three-charge system? Is more, less, or the same work required if q_3 is moved along a path that is (e) inside the rectangle but not on a diagonal and (f) outside the rectangle?



- *Thanks for your attention!*
- *Please review this chapter in time!*